EFFECTIVE FIELD THEORIES

FOR NON-PERTURBATIVE

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Motivation

Real Motivation: to understand the connection between non-relativistic (NR) Quantum Mechanics and Quantum Field Theories.

Better understanding of QCD and better determination of the parameters of the Standard Model: m_b, m_t, ...

"Physical Systems":

1) NR bound state systems:

- QED: positronium, Hydrogen-like/exotic atoms, atomic physics ...
- QCD: Heavy Quarkonium (Υ, J/ψ, B_c ...), Hybrids(?), nuclear physics ...

2) Q-$\bar{Q}$ production near threshold (t-$\bar{t}$ at NLC).

3) Static systems $\leftrightarrow$ lattice "experimental" data.

Tool: Effective Field Theories

Why?: There is a hierarchy of different scales (hard, soft and ultrasoft).

$$m >> mv >> mv^2 \,, \quad (\Lambda_{QCD})$$

EFTs are especially useful in these situations.

1) Perturbative calculations much easier and systematic.

2) Nonperturbative information is parameterized in a model independent way.

3) Power counting.

NR Effective Field Theories

Our aim is to provide a systematic method to deal with NR bound state systems. We will introduce a hierarchy of EFTs when sequentially integrating out each scale (only one scale in each step, strong simplification).

QCD

↓ Integrating out the hard scale (m)

NRQCD

↓ Integrating out the soft scale (mv)

pNRQCD

$$\left.\begin{array}{l}\left(i\partial_0 - \dfrac{\mathbf{p}^2}{2m} - V_0(r)\right)\Phi(\mathbf{r}) = 0 \\ +\text{corrections to the potential} \\ +\text{interaction with other low} \\ \quad\text{energy degrees of freedom}\end{array}\right\} \text{potential NRQCD}$$

In the perturbative case the starting point is $V_0 = -C_f \dfrac{\alpha}{r}$.

In the non-perturbative case?

NRQCD: the scale m

NRQCD has an ultraviolet cutoff Λ such that $m \gg \Lambda$ and larger than any other dynamical scale in the problem. $\Psi = \psi + \chi$

$$\mathcal{L}_{NRQCD} = \bar{\Psi} i\gamma^0 D_0 \Psi$$
$$+\bar{\Psi}\{\frac{\mathbf{D}^2}{2m} + c_F\, g\frac{\mathbf{\Sigma}\cdot\mathbf{B}}{2m} + c_D\, g\frac{\gamma^0\left(\mathbf{D}\cdot\mathbf{E}-\mathbf{E}\cdot\mathbf{D}\right)}{8m^2}$$
$$+ic_S\, g\frac{\gamma^0\mathbf{\Sigma}\cdot(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})}{8m^2} + \frac{\mathbf{D}^4}{8m^3}\}\Psi$$
$$-\frac{1}{4}d_1 F_{\mu\nu}F^{\mu\nu} + \frac{d_2}{m^2}F_{\mu\nu}D^2F^{\mu\nu} + \frac{d_3}{m^2}g f_{ABC}F^A_{\mu\nu}F^B_{\mu\alpha}F^C_{\nu\alpha}$$

$$\delta\mathcal{L}_{NRQCD} = \frac{d_{ss}}{m_1 m_2}\psi_1^\dagger\psi_1\chi_2^\dagger\chi_2 + \frac{d_{sv}}{m_1 m_2}\psi_1^\dagger\boldsymbol{\sigma}\psi_1\chi_2^\dagger\boldsymbol{\sigma}\chi_2$$
$$+\frac{d_{vs}}{m_1 m_2}\psi_1^\dagger \mathrm{T}^a\psi_1\chi_2^\dagger\mathrm{T}^a\chi_2 + \frac{d_{vv}}{m_1 m_2}\psi_1^\dagger\mathrm{T}^a\boldsymbol{\sigma}\psi_1\chi_2^\dagger\mathrm{T}^a\boldsymbol{\sigma}\chi_2\,.$$

Lepage, Caswell, Thacker

$c_i = 1 + O(\alpha_s)$, $d_1 = 1 + O(\alpha_s^2)$ **(we use the relevant α_s at low energies),** d_2 **,**d_3**,** d_{ss}**, ...**$= O(\alpha_s)$**.**

Typically,

$$c_i \sim 1 + \alpha_s\left(A\log\frac{m}{\mu}+B\right) \qquad d_i \sim \alpha_s\left(1+\alpha_s\left(A\log\frac{m}{\mu}+B\right)\right)$$

Manohar; Soto, Pineda

pNRQCD: the scale mv

The integration of the **mv** scale gives rise to potential terms. The Lagrangian is local in time but not in space.

Playing with the scales:

1) $mv \sim \Lambda_{QCD}$

2) $mv \gg \Lambda_{QCD} \gg mv^2$

3) $mv \gg mv^2 \sim \Lambda_{QCD}$

4) $mv \gg mv^2 \gg \Lambda_{QCD}$

Loosely speaking, when to trust the perturbative calculation and the size of NP corrections.

$\boxed{mv \sim \Lambda_{QCD}}$ (b-$\bar{b}$,c-$\bar{c}$)

- Degrees of freedom
- symmetries
- cutoff

pNRQCD has two ultraviolet cut-offs, ν_{us} and ν_p. ν_{us} fulfils the relation $\mathbf{p}^2/m \ll \nu_{us} \ll |\mathbf{p}|$ and is the cut-off of the energy of the quarks, and of the energy and the momentum of the gluons. ν_p fulfils $|\mathbf{p}| \ll \nu_p \ll m$ and is the cut-off of the relative momentum of the quark–antiquark system, $\mathbf{p}$.

Gauge transformation: Singlet: S, Octet: O

$$S(\mathbf{x}, \mathbf{X}, t) \to S(\mathbf{x}, \mathbf{X}, t)\,,$$
$$O(\mathbf{x}, \mathbf{X}, t) \to g(\mathbf{X}, t) O(\mathbf{x}, \mathbf{X}, t) g^{-1}(\mathbf{X}, t)\,.$$

Interpolating fields:

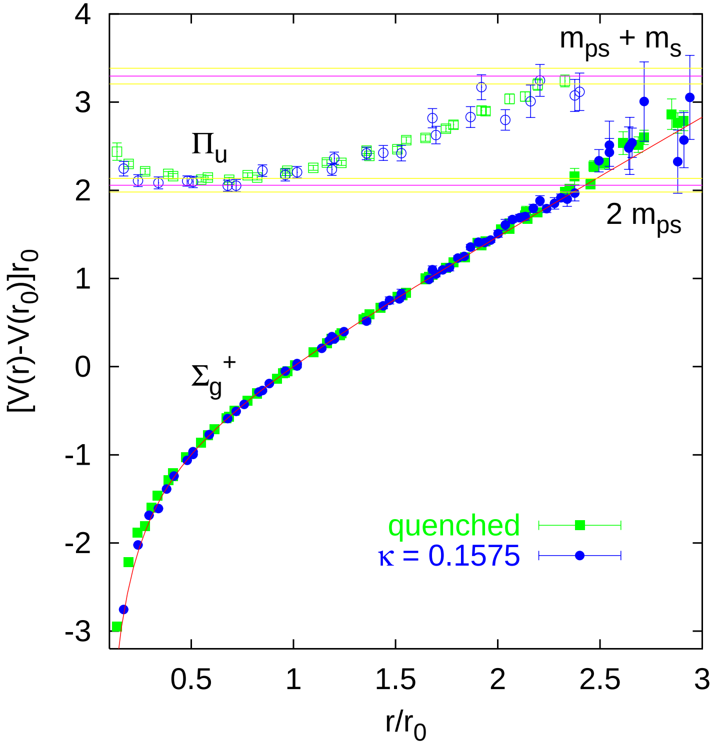
$$Q_2^\dagger(\mathbf{x}_2, t)\phi(\mathbf{x}_2, \mathbf{x}_1; t) Q_1(\mathbf{x}_1, t) = Z_s^{1/2}(\mathbf{x}) S(\mathbf{X}, \mathbf{x}, t)$$

$$Q_2^\dagger(x_2)\phi(\mathbf{x}_2, \mathbf{X}; t) T^a \phi(\mathbf{X}, \mathbf{x}_1; t) Q_1(x_1) = Z_o^{1/2}(\mathbf{x}) O^a(\mathbf{X}, \mathbf{x}, t)$$

$$\phi(\mathbf{y}, \mathbf{x}, t) \equiv \mathrm{P}\exp\left\{ig \int_0^1 ds\, (\mathbf{y} - \mathbf{x}) \cdot \mathbf{A}(\mathbf{x} - s(\mathbf{x} - \mathbf{y}), t)\right\}$$

$mv \sim \Lambda_{QCD}$ (b-$\bar{b}$,c-$\bar{c}$)

Degrees of freedom (?): (trial-error, mass gap, we do not see hybrids)
Static NRQCD: $D_{\infty h}$ **(substituting parity by CP).**



Matching scale $\nu_{us} \ll \Lambda_{QCD}$**. Coloured-like degrees of freedom decouple. Mass gap of hybrids and glueballs of** $O(\Lambda_{QCD}) \gg mv^2$**.**

S, O and soft gluons → S

$r_0 \simeq 0.5$ **fm. From SESAM, hep-lat/0003012.**

- **Pure QCD (no light fermions): the singlet (S)**
- **QCD: singlet plus pions (non-potential effects).**

Power counting/scales

Scales: m, p, $1/r$, Λ_{QCD}, mv^2,...

Dimensionless quantities:

$$\frac{p}{m},\ \alpha_s(m),\ \frac{1}{m\,r},\ \frac{\Lambda_{QCD}}{m},\ (mv^2)r \ll 1$$

pNRQCD Lagrangian

$$\mathcal{L}^{(0)} = S^\dagger \left(i\partial_0 - h_s(\mathbf{r},\mathbf{p}_1,\mathbf{p}_2)\right) S$$

$$h_s(\mathbf{x}_1,\mathbf{x}_2,\mathbf{p}_1,\mathbf{p}_2) = \frac{\mathbf{p}_1^2}{2m_1} + \frac{\mathbf{p}_2^2}{2m_2} + V^{(0)} + \frac{V^{(1,0)}}{m_1} + \frac{V^{(0,1)}}{m_2} + \frac{V^{(2,0)}}{m_1^2} + \frac{V^{(0,2)}}{m_2^2} + \frac{V^{(1,1)}}{m_1 m_2}.$$

$$V^{(2,0)} = \frac{1}{2}\left\{\mathbf{p}_1^2, V_{\mathbf{p}^2}^{(2,0)}(r)\right\} + \frac{V_{\mathbf{L}^2}^{(2,0)}(r)}{r^2}\mathbf{L}_1^2 + V_r^{(2,0)}(r) \quad \text{(SI)}$$
$$+V_{LS}^{(2,0)}(r)\mathbf{L}_1\cdot\mathbf{S}_1 \quad \text{(SD)}$$

$$V^{(1,1)} = -\frac{1}{2}\left\{\mathbf{p}_1\cdot\mathbf{p}_2, V_{\mathbf{p}^2}^{(1,1)}(r)\right\} - \frac{V_{\mathbf{L}^2}^{(1,1)}(r)}{2r^2}(\mathbf{L}_1\cdot\mathbf{L}_2 + \mathbf{L}_2\cdot\mathbf{L}_1) + V_r^{(1,1)}(r) \quad \text{(SI)}$$
$$+V_{L_1S_2}^{(1,1)}(r)\mathbf{L}_1\cdot\mathbf{S}_2 - V_{L_2S_1}^{(1,1)}(r)\mathbf{L}_2\cdot\mathbf{S}_1 + V_{S^2}^{(1,1)}(r)\mathbf{S}_1\cdot\mathbf{S}_2 + V_{\mathbf{S}_{12}}^{(1,1)}(r)\mathbf{S}_{12}(\hat{\mathbf{r}}) \quad \text{(SD)}$$

where $\mathbf{S}_{12}(\hat{\mathbf{r}}) \equiv 3\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_1\,\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$.

Matching

Expansion in $1/M$. HQET can be used in the matching (static sources).

Wilson loop formalism. Suitable if we can not work perturbatively. For example, the computation of the following Green function in both theories

$$\langle 0|Q_2^\dagger(x_2)\phi(x_2,x_1)Q_1(x_1)Q_1^\dagger(y_1)\phi(y_1,y_2)Q_2(y_2)|0\rangle,$$

NRQCD

$$\delta^3(\mathbf{x}_1-\mathbf{y}_1)\delta^3(\mathbf{x}_2-\mathbf{y}_2)\langle W_\Box\rangle,$$

pNRQCD

$$Z_s(\mathbf{r})\delta^3(\mathbf{x}_1-\mathbf{y}_1)\delta^3(\mathbf{x}_2-\mathbf{y}_2)e^{-iTV_s^{(0)}(\mathbf{r})}$$

One obtains:

$$V^{(0)}(\mathbf{r}) = \lim_{T\to\infty}\frac{i}{T}\log\langle W_\Box\rangle \quad \text{Wilson, Susskind}$$

No complete/correct method to obtain the subleading potentials in $1/m$ in the past.

$V_{LS}^{(2,0)}$, $V_{S^2}^{(1,1)}$, $V_{S_{12}}^{(1,1)}$: **Eichten, Feinberg; Peskin; Gromes; Cheng, Kuang, Oakes; Barchielli, Montaldi, Prosperi; Barchielli, Brambilla, Prosperi; Szczepaniak, Swanson; Vairo, Pineda**

$V_{L_1S_2}^{(1,1)}$: **Vairo, Pineda; Szczepaniak, Swanson**

$V_{\mathbf{p}^2}^{(2,0)}$, $V_{\mathbf{L}^2}^{(2,0)}$, $V_{\mathbf{p}^2}^{(1,1)}$, $V_{\mathbf{L}^2}^{(1,1)}$: **Barchielli, Montaldi, Prosperi; Barchielli, Brambilla, Prosperi; Vairo, Pineda**

$V^{(1,1)}$: **Brambilla, Soto, Vairo, Pineda**

$V_r^{(2,0)}$, $V_r^{(1,1)}$: **Vairo, Pineda**

Missing/wrong terms in the literature!! until recently.

We have provided the method to compute the potential in terms of Wilson loops at arbitrary order in $O(1/M)$. Either by standard matching or by a quantum mechanics computation-like in the $1/M$ expansion. Brambilla, Soto, Vairo, Pineda

We have reduced (but not eliminated) the model dependence. The potentials can be computed in the lattice or with models.

We have made quantitative the relation between NRQCD and potential models. OK

Challenge:

- **Incorporate pions: Remains to be done.**
- **Clarify the power counting. Only some partial results.**

Hamiltonian Formalism

Static limit: $H_{NRQCD} \simeq H^{(0)}$

$$|\underline{\mathbf{n}}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} \equiv \psi^\dagger(\mathbf{x}_1)\chi_c^\dagger(\mathbf{x}_2)|n; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}, \qquad E_n^{(0)}(r)$$

$$\left.\begin{array}{l} |\Sigma_g^+; \mathbf{r}\rangle^{(0)} \\ |\Pi; \mathbf{r}\rangle^{(0)} \\ |\Delta; \mathbf{r}\rangle^{(0)} \\ \dots \end{array}\right\} \text{Spectrum in the static limit}$$

Σ_g^+. **Ground state of NRQCD in the static limit.**

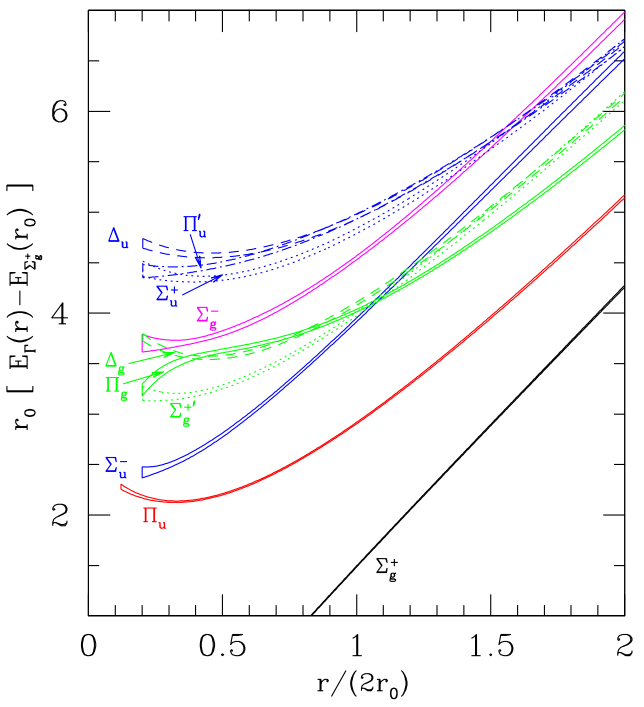


Figure 1: *Energies for different gluonic excitation between static quarks at distance r from the quenched lattice measurements of Juge et al. hep-lat/9809015, $r_0 \simeq 0.5$ fm. The picture is taken from Michael, hep-ph/9809211.*

$$H_{NRQCD} = H^{(0)} + \frac{1}{m}H^{(1)} + \frac{1}{m^2}H^{(2)} + O(1/m^3)$$

$$|\underline{\mathbf{n}}; \mathbf{x}_1, \mathbf{x}_2\rangle, \qquad E_n(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2)$$

Matching condition:

$$\boxed{S|0\rangle \equiv |\underline{\mathbf{0}}\rangle, \quad E_0(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2) = h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2)}$$

$O(1/m^0)$

$$\boxed{V_s^{(0)}(r) = E_{\Sigma_g^+}(r)}$$

$O(1/m)$

$$\left[\frac{\mathbf{p}^2}{2} + V^{(1)}(r)\right]\delta^{(3)}(\mathbf{x}_1 - \mathbf{y}_1)\delta^{(3)}(\mathbf{x}_2 - \mathbf{y}_2)$$
$$= {}^{(0)}\langle \underline{\mathbf{0}}; \mathbf{x}_1, \mathbf{x}_2 | H^{(1)} | \underline{\mathbf{0}}; \mathbf{y}_1, \mathbf{y}_2 \rangle^{(0)}$$
$$= \left(-\frac{\boldsymbol{\nabla}_{\mathbf{x}_1}{}^2}{2} + \frac{1}{2} \sum_{n \neq 0} \left| \frac{{}^{(0)}\langle n | g\mathbf{E} | 0 \rangle^{(0)}}{E_0^{(0)} - E_n^{(0)}} \right|^2 \right)$$
$$\times \delta^{(3)}(\mathbf{x}_1 - \mathbf{y}_1)\delta^{(3)}(\mathbf{x}_2 - \mathbf{y}_2).$$

We have used

a) $${}^{(0)}\langle n | \mathbf{D}_{\mathbf{x}_1} | n \rangle^{(0)} = \boldsymbol{\nabla}_{\mathbf{x}_1},$$

b) $${}^{(0)}\langle n | \mathbf{D}_{\mathbf{x}_1} | j \rangle^{(0)} = \frac{{}^{(0)}\langle n | g\mathbf{E}(\mathbf{x}_1) | j \rangle^{(0)}}{E_n^{(0)} - E_j^{(0)}} \quad \forall\, n \neq j,$$

Finally

$$V^{(1)}(r) = \frac{1}{2} \sum_{n \neq 0} \left| \frac{{}^{(0)}\langle n | g\mathbf{E} | 0 \rangle^{(0)}}{E_0^{(0)} - E_n^{(0)}} \right|^2 .$$

$$V^{(1,1)} = -\frac{1}{2}\lim_{T\to\infty}\int_0^T dt\, t\, \langle\!\langle g\mathbf{E}_1(t)\cdot g\mathbf{E}_1(0)\rangle\!\rangle_c.$$

$$V^{(2,0)}_{\mathbf{p}^2}(r) = \frac{i}{2}\hat{\mathbf{r}}^i\hat{\mathbf{r}}^j \lim_{T\to\infty}\int_0^T dt\, t^2\langle\!\langle g\mathbf{E}^i_1(t) g\mathbf{E}^j_1(0)\rangle\!\rangle_c,$$

$$V^{(2,0)}_{\mathbf{L}^2}(r) = \frac{i}{4}\left(\delta^{ij} - 3\hat{\mathbf{r}}^i\hat{\mathbf{r}}^j\right) \lim_{T\to\infty}\int_0^T dt\, t^2\langle\!\langle g\mathbf{E}^i_1(t) g\mathbf{E}^j_1(0)\rangle\!\rangle_c,$$

$$\begin{aligned}
V^{(2,0)}_r(r) = &-\frac{c_D^{(1)}}{8}\lim_{T\to\infty}\int_0^T dt\, \langle\!\langle [\mathbf{D}_1, g\mathbf{E}_1(t)]\rangle\!\rangle_c \\
&-\frac{i c_F^{(1)\,2}}{4}\lim_{T\to\infty}\int_0^T dt\, \langle\!\langle g\mathbf{B}_1(t)\cdot g\mathbf{B}_1(0)\rangle\!\rangle_c \\
&-\frac{i}{2}\lim_{T\to\infty}\int_0^T dt_1\int_0^{t_1} dt_2\int_0^{t_2} dt_3\, (t_2 - t_3)^2 \\
&\qquad\times\langle\!\langle g\mathbf{E}_1(t_1)\cdot g\mathbf{E}_1(t_2) g\mathbf{E}_1(t_3)\cdot g\mathbf{E}_1(0)\rangle\!\rangle_c \\
&+\frac{1}{2}(\boldsymbol{\nabla}^i_r \lim_{T\to\infty}\int_0^T dt_1\int_0^{t_1} dt_2\, (t_1 - t_2)^2 \\
&\qquad\times\langle\!\langle g\mathbf{E}^i_1(t_1) g\mathbf{E}_1(t_2)\cdot g\mathbf{E}_1(0)\rangle\!\rangle_c) \\
&-\frac{i}{2}\left(\boldsymbol{\nabla}^i_r V^{(0)}\right)\lim_{T\to\infty}\int_0^T dt_1\int_0^{t_1} dt_2\, (t_1 - t_2)^3 \\
&\qquad\times\langle\!\langle g\mathbf{E}^i_1(t_1) g\mathbf{E}_1(t_2)\cdot g\mathbf{E}_1(0)\rangle\!\rangle_c \\
&-\frac{1}{2}\lim_{T\to\infty}\int_0^T dt_1\int_0^{t_1} dt_2\, (t_1 - t_2)^2
\end{aligned}$$

$$
\begin{aligned}
&\times \langle\langle [\mathbf{D}_{1}\cdot, g\mathbf{E}_1](t_1) g\mathbf{E}_1(t_2)\cdot g\mathbf{E}_1(0)\rangle\rangle_c \\
&+\frac{i}{8}\lim_{T\to\infty}\int_0^T dt\, t^2 \langle\langle [\mathbf{D}_{1}\cdot, g\mathbf{E}_1](t)[\mathbf{D}_{1}\cdot, g\mathbf{E}_1](0)\rangle\rangle_c \\
&-\frac{i}{4}(\boldsymbol{\nabla}_r^i \lim_{T\to\infty}\int_0^T dt\, t^2 \langle\langle g\mathbf{E}_1^i(t)[\mathbf{D}_{1}\cdot, g\mathbf{E}_1](0)\rangle\rangle_c) \\
&-\frac{1}{4}\lim_{T\to\infty}\int_0^T dt\, t^3 \langle\langle [\mathbf{D}_{1}\cdot, g\mathbf{E}_1](t) g\mathbf{E}_1^j(0)\rangle\rangle_c (\boldsymbol{\nabla}_r^j V^{(0)}) \\
&+\frac{1}{4}(\boldsymbol{\nabla}_r^i \lim_{T\to\infty}\int_0^T dt\, t^3 \langle\langle g\mathbf{E}_1^i(t) g\mathbf{E}_1^j(0)\rangle\rangle_c (\boldsymbol{\nabla}_r^j V^{(0)})) \\
&+\frac{1}{2}(\boldsymbol{\nabla}_r^2 V_{\mathbf{p}^2}^{(2,0)}) - \frac{i}{12}\lim_{T\to\infty}\int_0^T dt\, t^4 \\
&\times \langle\langle g\mathbf{E}_1^i(t) g\mathbf{E}_1^j(0)\rangle\rangle_c (\boldsymbol{\nabla}_r^i V^{(0)})(\boldsymbol{\nabla}_r^j V^{(0)}) \\
&-d_3'^{(1)} f_{abc}\int d^3\mathbf{x} \lim_{T_W\to\infty} g\langle\langle G^a_{\mu\nu}(x) G^b_{\mu\alpha}(x) G^c_{\nu\alpha}(x)\rangle\rangle,
\end{aligned}
$$

$$
\begin{aligned}
V_{LS}^{(2,0)}(r) &= \frac{c_S^{(1)}}{2r^2}\mathbf{r}\cdot(\boldsymbol{\nabla}_r V^{(0)}) \\
&-\frac{c_F^{(1)}}{r^2} i\mathbf{r}\cdot \lim_{T\to\infty}\int_0^T dt\, t\, \langle\langle g\mathbf{B}_1(t)\times g\mathbf{E}_1(0)\rangle\rangle.
\end{aligned}
$$

$$
V_{\mathbf{p}^2}^{(1,1)}(r) = i\hat{\mathbf{r}}^i\hat{\mathbf{r}}^j \lim_{T\to\infty}\int_0^T dt\, t^2 \langle\langle g\mathbf{E}_1^i(t) g\mathbf{E}_2^j(0)\rangle\rangle_c,
$$

$$
V_{\mathbf{L}^2}^{(1,1)}(r) = i\frac{\delta^{ij}-3\hat{\mathbf{r}}^i\hat{\mathbf{r}}^j}{2}\lim_{T\to\infty}\int_0^T dt\, t^2 \langle\langle g\mathbf{E}_1^i(t) g\mathbf{E}_2^j(0)\rangle\rangle_c,
$$

$$
\begin{aligned}
V_r^{(1,1)}(r) &= -\frac{1}{2}(\boldsymbol{\nabla}_r^2 V_{\mathbf{p}^2}^{(1,1)}) \\
&-i \lim_{T\to\infty} \int_0^T dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3\, (t_2 - t_3)^2 \\
&\qquad \times \langle\langle g\mathbf{E}_1(t_1)\cdot g\mathbf{E}_1(t_2) g\mathbf{E}_2(t_3)\cdot g\mathbf{E}_2(0)\rangle\rangle_c \\
&+\frac{1}{2}(\boldsymbol{\nabla}_r^i \lim_{T\to\infty} \int_0^T dt_1 \int_0^{t_1} dt_2 (t_1-t_2)^2 \langle\langle g\mathbf{E}_1^i(t_1) g\mathbf{E}_2(t_2)\cdot g\mathbf{E}_2(0)\rangle\rangle_c) \\
&+\frac{1}{2}(\boldsymbol{\nabla}_r^i \lim_{T\to\infty} \int_0^T dt_1 \int_0^{t_1} dt_2 (t_1-t_2)^2 \langle\langle g\mathbf{E}_2^i(t_1) g\mathbf{E}_1(t_2)\cdot g\mathbf{E}_1(0)\rangle\rangle_c) \\
&-\frac{i}{2}\left(\boldsymbol{\nabla}_r^i V^{(0)}\right) \lim_{T\to\infty} \int_0^T dt_1 \int_0^{t_1} dt_2 (t_1-t_2)^3 \\
&\qquad \times \langle\langle g\mathbf{E}_1^i(t_1) g\mathbf{E}_2(t_2)\cdot g\mathbf{E}_2(0)\rangle\rangle_c \\
&-\frac{i}{2}\left(\boldsymbol{\nabla}_r^i V^{(0)}\right) \lim_{T\to\infty} \int_0^T dt_1 \int_0^{t_1} dt_2 (t_1-t_2)^3 \\
&\qquad \times \langle\langle g\mathbf{E}_2^i(t_1) g\mathbf{E}_1(t_2)\cdot g\mathbf{E}_1(0)\rangle\rangle_c \\
&-\frac{1}{2} \lim_{T\to\infty} \int_0^T dt_1 \int_0^{t_1} dt_2 (t_1-t_2)^2 \langle\langle [\mathbf{D}_1 ., g\mathbf{E}_1](t_1) g\mathbf{E}_2(t_2)\cdot g\mathbf{E}_2(0)\rangle\rangle_c \\
&+\frac{1}{2} \lim_{T\to\infty} \int_0^T dt_1 \int_0^{t_1} dt_2 (t_1-t_2)^2 \langle\langle [\mathbf{D}_2 ., g\mathbf{E}_2](t_1) g\mathbf{E}_1(t_2)\cdot g\mathbf{E}_1(0)\rangle\rangle_c \\
&-\frac{i}{4} \lim_{T\to\infty} \int_0^T dt\, t^2 \langle\langle [\mathbf{D}_1 ., g\mathbf{E}_1](t) [\mathbf{D}_2 ., g\mathbf{E}_2](0)\rangle\rangle_c \\
&+\frac{i}{4}(\boldsymbol{\nabla}_r^i \lim_{T\to\infty} \int_0^T dt\, t^2 \{ \langle\langle g\mathbf{E}_1^i(t) [\mathbf{D}_2 ., g\mathbf{E}_2](0)\rangle\rangle_c
\end{aligned}
$$

$$-\langle\!\langle g\mathbf{E}_2^i(t)[\mathbf{D}_1\cdot, g\mathbf{E}_1](0)\rangle\!\rangle_c\}\Big)$$

$$-\frac{1}{4}\lim_{T\to\infty}\int_0^T dt\, t^3\{\langle\!\langle[\mathbf{D}_1\cdot, g\mathbf{E}_1](t)g\mathbf{E}_2^j(0)\rangle\!\rangle_c$$

$$-\langle\!\langle[\mathbf{D}_2\cdot, g\mathbf{E}_2](t)g\mathbf{E}_1^j(0)\rangle\!\rangle_c\}(\boldsymbol{\nabla}_r^j V^{(0)})$$

$$+\frac{1}{4}(\boldsymbol{\nabla}_r^i\lim_{T\to\infty}\int_0^T dt\, t^3\{\langle\!\langle g\mathbf{E}_1^i(t)g\mathbf{E}_2^j(0)\rangle\!\rangle_c$$

$$+\langle\!\langle g\mathbf{E}_2^i(t)g\mathbf{E}_1^j(0)\rangle\!\rangle_c\}(\boldsymbol{\nabla}_r^j V^{(0)}))$$

$$-\frac{i}{6}\lim_{T\to\infty}\int_0^T dt\, t^4\langle\!\langle g\mathbf{E}_1^i(t)g\mathbf{E}_2^j(0)\rangle\!\rangle_c(\boldsymbol{\nabla}_r^i V^{(0)})(\boldsymbol{\nabla}_r^j V^{(0)})$$

$$+(d_{ss}+d_{vs}\lim_{T_W\to\infty}\langle\!\langle T_1^a T_2^a\rangle\!\rangle)\,\delta^{(3)}(\mathbf{x}_1-\mathbf{x}_2),$$

$$V_{L_2S_1}^{(1,1)}(r) = -\frac{c_F^{(1)}}{r^2}i\mathbf{r}\cdot\lim_{T\to\infty}\int_0^T dt\, t\,\langle\!\langle g\mathbf{B}_1(t)\times g\mathbf{E}_2(0)\rangle\!\rangle,$$

$$V_{S^2}^{(1,1)}(r) = \frac{2c_F^{(1)}c_F^{(2)}}{3}i\lim_{T\to\infty}\int_0^T dt\,\langle\!\langle g\mathbf{B}_1(t)\cdot g\mathbf{B}_2(0)\rangle\!\rangle$$

$$-4(d_{sv}+d_{vv}\lim_{T_W\to\infty}\langle\!\langle T_1^a T_2^a\rangle\!\rangle)\,\delta^{(3)}(\mathbf{x}_1-\mathbf{x}_2),$$

$$V_{\mathbf{S}_{12}}^{(1,1)}(r) = \frac{c_F^{(1)}c_F^{(2)}}{4}i\hat{\mathbf{r}}^i\hat{\mathbf{r}}^j\lim_{T\to\infty}\int_0^T dt$$

$$\times[\langle\!\langle g\mathbf{B}_1^i(t)\, g\mathbf{B}_2^j(0)\rangle\!\rangle - \frac{\delta^{ij}}{3}\langle\!\langle g\mathbf{B}_1(t)\cdot g\mathbf{B}_2(0)\rangle\!\rangle].$$

Comparison with Eichten–Feinberg spin-orbit potential

$$
\begin{aligned}
V^{(1,1)}_{L_2S_1}(r) &= -\frac{i}{r^2}\mathbf{r}\cdot \lim_{T\to\infty}\int_0^T dt\, t\, \langle\!\langle g\mathbf{B}_1(t)\times g\mathbf{E}_2(0)\rangle\!\rangle \\
&= \frac{i}{r^2}\mathbf{r}\cdot\sum_{k\neq 0}\frac{{}^{(0)}\langle 0|g\mathbf{B}_1|k\rangle^{(0)}\times {}^{(0)}\langle k|g\mathbf{E}_2^T|0\rangle^{(0)}}{(E_0^{(0)}-E_k^{(0)})^2} \\
&\underset{\text{pert}}{=} \frac{C_f\alpha_s}{r^3}+O(\alpha_s^2).
\end{aligned}
$$

Eichten and Feinberg obtain:

$$
\begin{aligned}
&V^{(1,1)}_{L_2S_1}(r) \\
&= \frac{i}{2r^2}\lim_{T_W\to\infty}\frac{1}{T_W}\int_{-T_W/2}^{T_W/2}dt\,\int_{-T_W/2}^{T_W/2}dt'\,t' \\
&\qquad \times\mathbf{r}\cdot\langle\!\langle g\mathbf{B}_1(t)\times g\mathbf{E}_2(t')\rangle\!\rangle \\
&= \frac{i}{2r^2}\mathbf{r}\cdot\sum_{k\neq 0}\frac{{}^{(0)}\langle 0|g\mathbf{B}_1|k\rangle^{(0)}\times {}^{(0)}\langle k|g\mathbf{E}_2^T|0\rangle^{(0)}}{(E_0^{(0)}-E_k^{(0)})^2} \\
&\quad +\text{end}-\text{point string dependent terms} \\
&\underset{\text{pert}}{=} \frac{C_f\alpha_s}{2r^3}+O(\alpha_s^2),
\end{aligned}
$$

Power counting

Standard NRQCD power counting: based upon perturbation theory.

Less restrictive possibilities. Naturalness. The dynamics of the heavy quark is now explicit.

$V^{(0)}$

naturalness: $V^{(0)} \sim mv$

Virial theorem: $V^{(0)} \sim mv^2$

Perturbation theory: $V^{(0)} \sim mv\alpha_s$.

$V^{(1)}$

naturalness: $V^{(1)} \sim mv^2$ **!?**

Perturbation theory: $V^{(1)} \sim mv^2\alpha_s^2$.

$V^{(2)}$

naturalness: $V^{(2)} \sim mv^3$

Perturbation theory: $V^{(2)} \sim mv^3\alpha_s$.

Further constraints:

Virial Theorem. Terms involving $\boldsymbol{\nabla} V^{(0)} \sim mv^3$ are suppressed by an extra factor of v.

Gromes relation:

$$\frac{1}{2r}\frac{dV^{(0)}}{dr} + V_{LS}^{(2,0)} - V_{L_2S_1}^{(1,1)} = 0,$$

implies

$$V_{LS}^{(2,0)} - V_{L_2S_1}^{(1,1)} \sim mv^4$$

INCLUSIVE DECAYS

$$\Gamma = -2 \langle n, L, S, J | \mathrm{Im}\, h | n, L, S, J \rangle,$$

$\mathrm{Im}\, h$ **inherited from NRQCD four-fermion matching coefficients → Local potentials.**

$$\begin{aligned}\mathrm{Im}\, h \sim\ & \mathrm{Im}\, c_{4-f}^{d=6} \frac{\delta^{(3)}(\mathbf{r})}{m^2} \left(A + B \frac{\Lambda_{QCD}^2}{m^2} + \cdots \right) \\ & + \mathrm{Im}\, c_{4-f}^{d=8} \left(A' \frac{\{\delta^{(3)}(\mathbf{r}), \boldsymbol{\nabla}^2\}}{m^4} + B' \frac{\delta^{(3)}(\mathbf{x})}{m^2} \frac{\Lambda_{QCD}^2}{m^2} + \dots \right) \\ & + \mathrm{Im}\, c_{4-f}^{d=8} \left(C \mathcal{T}_{SJ}^{ij} \frac{\boldsymbol{\nabla}^i \delta^{(3)}(\mathbf{r}) \boldsymbol{\nabla}^j}{m^4} + \dots \right) + \dots ,\end{aligned}$$

S-wave decays

$$\begin{aligned}\Gamma \sim\ & \mathrm{Im}\, c_{4-f}^{d=6} \frac{|R_{ns0s}(0)|^2}{m^2} \left(A + B \frac{\Lambda_{QCD}^2}{m^2} + \cdots \right) \\ & + \mathrm{Im}\, c_{4-f}^{d=8} \left(A' \frac{R_{ns0s}(0)(\boldsymbol{\nabla}^2 R_{n10s}(0))}{m^4} + B' \frac{|R_{ns0s}(0)|^2}{m^2} \frac{\Lambda_{QCD}^2}{m^2} + \dots \right) + \dots ,\end{aligned}$$

P-wave decays

$$\Gamma \sim \mathrm{Im}\, c_{4-f}^{d=8} \left(C \frac{|\boldsymbol{\nabla} R_{nj1s}(0)|^2}{m^4} + \dots \right) \dots ,$$

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Leading contribution:

$$-2\,\mathrm{Im}\,h|_{\mathrm{P-wave}} = F_{SJ}\mathcal{T}^{ij}_{SJ}\frac{\boldsymbol{\nabla}^i_{\mathbf{r}}\delta^{(3)}(\mathbf{r})\boldsymbol{\nabla}^j_{\mathbf{r}}}{m^4}\,,$$

$$F_{SJ} = -2N_c\,\mathrm{Im}\,f_1({}^{2S+1}P_J) - \frac{4T_F}{9N_c}\,\mathcal{E}\;\mathrm{Im}\,f_8({}^{2S+1}S_S),$$

$$\mathcal{E} = T_F\int_0^\infty d\tau\,\tau^3\left\langle g\mathbf{E}^a(\tau,\mathbf{0})\Phi_{ab}(\tau,0;\mathbf{0})g\mathbf{E}^b(\tau,\mathbf{0})\right\rangle.$$

Up to $\mathcal{O}(1/m^4)$ the imaginary contributions are only carried by the matching coefficients of the dimension 6 and 8 4-heavy-fermion operators in NRQCD. Since we are only interested in P-wave decays a huge simplification occurs and only two contributions survive. From the dimension 8 operators the contribution reads

$$\mathrm{Im}\,\delta h\,\delta^{(3)}(\mathbf{x}_1-\mathbf{x}_1')\delta^{(3)}(\mathbf{x}_2-\mathbf{x}_2') = \frac{1}{m^4}\,\mathrm{Im}\,{}^{(0)}\langle\underline{0};\mathbf{x}_1,\mathbf{x}_2|H^{(4)}_{4-f}|\underline{0};\mathbf{x}_1',\mathbf{x}_2'\rangle^{(0)}|_{\mathrm{P-wave}}$$

$$= N_c\,\mathcal{T}^{ij}_{SJ}\,\mathrm{Im}\,f_1({}^{2S+1}P_J)\,\frac{\boldsymbol{\nabla}^i_{\mathbf{r}}\delta^{(3)}(\mathbf{r})\boldsymbol{\nabla}^j_{\mathbf{r}}}{m^4}\delta^{(3)}(\mathbf{x}_1-\mathbf{x}_1')\delta^{(3)}(\mathbf{x}_2-\mathbf{x}_2').$$

On the other hand, we also have contributions from the iteration of lower order $1/m$ corrections to the NRQCD Hamiltonian with the dimension 6 4-fermion operators. The only term that contributes reads

$$\mathrm{Im}\,\delta h\,\delta^{(3)}(\mathbf{x}_1-\mathbf{x}_1')\delta^{(3)}(\mathbf{x}_2-\mathbf{x}_2') = \frac{1}{m^4}\,\mathrm{Im}\,{}^{(1)}\langle\underline{0};\mathbf{x}_1,\mathbf{x}_2|H^{(2)}_{4-f}|\underline{0};\mathbf{x}_1',\mathbf{x}_2'\rangle^{(1)}|_{\mathrm{P-wave}}$$

$$= \frac{2T_F}{9N_c}\mathcal{E}\,\frac{\boldsymbol{\nabla}_{\mathbf{r}}\delta^{(3)}(\mathbf{r})\boldsymbol{\nabla}_{\mathbf{r}}}{m^4}\mathcal{T}_S\,\mathrm{Im}f_8({}^{2S+1}\mathrm{S}_S)\delta^{(3)}(\mathbf{x}_1-\mathbf{x}_1')\delta^{(3)}(\mathbf{x}_2-\mathbf{x}_2')$$

An infinity set of new relations can be obtained for the decay widths!!

$$\Gamma(\chi_{QJ}^{S}(n\mathrm{P}) \to \mathrm{LH}) = [\, \frac{3N_c}{\pi}\, \mathrm{Im}\, [f_1({}^{2S+1}\mathrm{P}_J)] + \frac{2T_F}{3\pi N_c} \mathrm{Im}\, [f_8({}^{2S+1}\mathrm{S}_S)]\, \mathcal{E}\,]\, \frac{|R'_{Qn1}(0)|^2}{m^4},$$

$$\boxed{\langle \chi_{QJ}^{S}(n\mathrm{P}) | O_8({}^1\mathrm{S}_0) | (\chi_{QJ}^{S}(n\mathrm{P}) \rangle(\mu) = \frac{|R'_{Qn1}(0)|^2}{6\pi N_c m^2} \mathcal{E}(\mu).}$$

Right scale dependence

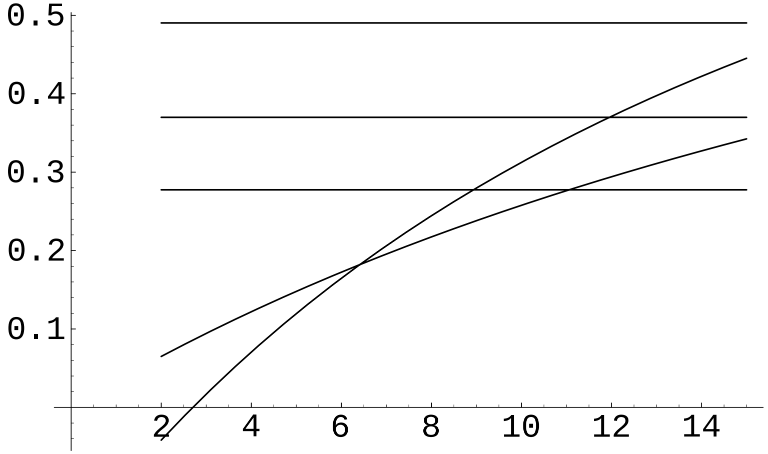
$$\mathcal{E} \simeq 12\, C_F N_c \frac{\alpha_s}{\pi}\, \ln \mu,$$

From Charmonium data

$$\mathcal{E}(1\,\mathrm{GeV}) \simeq 5.$$

$$\frac{\Gamma(\chi_{b1}^{1}(1\mathrm{P}))}{\Gamma(\chi_{b2}^{1}(1\mathrm{P}))} = \frac{\Gamma(\chi_{b1}^{1}(2\mathrm{P}))}{\Gamma(\chi_{b2}^{1}(2\mathrm{P}))} \simeq 0.5\,.$$

The other branching ratios can be obtained from this result plus using spin symmetry.



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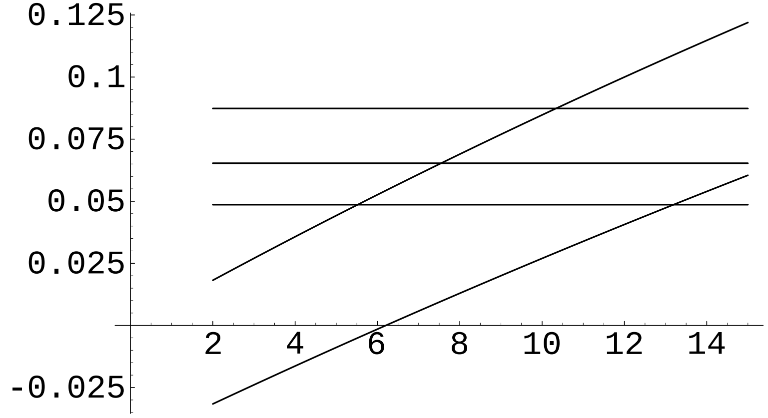
0.4
0.3
0.2
0.1
2
4
6
8
10
12
14



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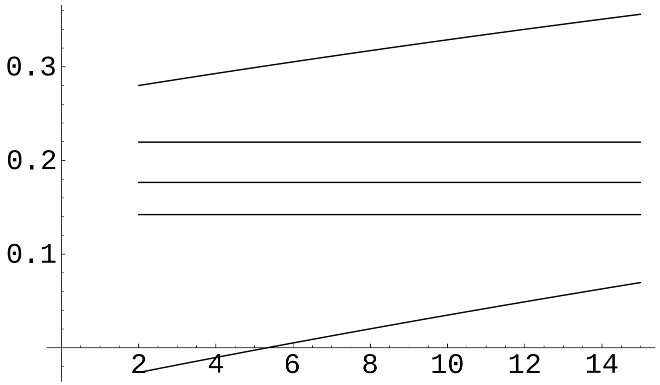
0.1
0.075
0.05
0.025
-0.025
2
4
6
8
10
12
14



0.3
0.2
0.1
2
4
6
8
10
12
14

NRQCD matrix elements

Master equation:

$$\langle H|O(\mathbf{0})|H\rangle = \int d^3\mathbf{r} \int d^3\mathbf{r}' \int d^3\mathbf{R} \int d^3\mathbf{R}' \langle \mathbf{P}=0|\mathbf{R}\rangle\langle njls|\mathbf{r}\rangle$$
$$\times[\langle \underline{\mathbf{0}};\mathbf{x}_1\mathbf{x}_2|O(\mathbf{0})\ |\underline{\mathbf{0}};\mathbf{x}_1'\mathbf{x}_2'\rangle]\langle \mathbf{R}'|\mathbf{P}=0\rangle\langle \mathbf{r}'|njls\rangle\,.$$

Examples:

$$\langle V_Q(nS)|O_1(^3S_1)|V_Q(nS)\rangle = C_A\frac{|R^V_{n0}(\mathbf{0})|^2}{2\pi}\left(1-\frac{E^{(0)}_{n0}}{m}\frac{2\mathcal{E}_3}{9}+\frac{2\mathcal{E}^{(2,t)}_3}{3m^2}+\frac{c_F^2\mathcal{B}_1}{3m^2}\right),$$

$$\langle V_Q(nS)|\mathcal{P}_1(^3S_1)|V_Q(nS)\rangle = \langle P_Q(nS)|\mathcal{P}_1(^1S_0)|P_Q(nS)\rangle = C_A\frac{|R^{(0)}_{n0}(\mathbf{0})|^2}{2\pi}\left(mE^{(0)}_{n0}-\mathcal{E}_1\right),$$

$$\langle V_Q(nS)|O_8(^3S_1)|V_Q(nS)\rangle = \langle P_Q(nS)|O_8(^1S_0)|P_Q(nS)\rangle$$
$$= C_A\frac{|R^{(0)}_{n0}(\mathbf{0})|^2}{2\pi}\left(-\frac{2(C_A/2-C_f)\mathcal{E}^{(2)}_3}{3m^2}\right),$$

$$\langle V_Q(nS)|O_8(^1S_0)|V_Q(nS)\rangle = \frac{\langle P_Q(nS)|O_8(^3S_1)|P_Q(nS)\rangle}{3}$$
$$= C_A\frac{|R^{(0)}_{n0}(\mathbf{0})|^2}{2\pi}\left(-\frac{(C_A/2-C_f)c_F^2\mathcal{B}_1}{3m^2}\right).$$

Number of independent constants is reduced: $\boxed{46\to 19}$ **Wave-function at the origen and non-local gluonic correlators:** $\mathcal{E}_n$, $\mathcal{B}_n$, ...

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Conclusions

We have studied the application of EFT's to NP Heavy Quarkonium. In particular we have studied a new theory: pNRQCD with only ultrasoft degrees of freedom.

It is now cleaner under which circumstances a pure Schrödinger-like formulation (potential model) is valid in the nonperturbative regime ($mv \sim \Lambda_{QCD}$) and how it can be derived in a controlled manner from QCD.

- **It is now possible to perform a well defined expansion at any order in $1/m$ in order to obtain the potential in the non-perturbative regime of any gluonic excitation.**
- **The heavy quarkonium potential is known at $O(1/m^2)$ in terms of Wilson loops. The potential of any gluonic excitation is also known in terms of matrix elements of the static solution. We have obtained new potentials previously missed in the literature and also corrected others.**
- **We have shown that even if working with a pure Schrödinger-like formulation (potential model) the effects due to NRQCD color-octet matrix elements are incorporated in our formalism.**
- **The factorization between the soft and ultrasoft scale achieved with pNRQCD allows to obtain a new set of non-trivial predictions for the inclusive heavy quarkonium P-wave decays.**