Breakdown of the operator product expansion in the 't Hooft model

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Outline

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Motivation

We would like to have a model-independent and analytic approach to QCD.

Nowdays confinement is beyond reach. Therefore, in principle, the only model-independent and analytic approach to QCD is perturbation theory and the OPE (perturbative factorization) but ...

Degree of rigour:

1) Quark-hadron duality. Computations in the Minkowski (physical) cut.

2) Validity of the OPE itself. So far only proven in perturbation theory.

Let us first set the discussion for the vector-vector correlator

$$(q^\mu q^\nu - g^{\mu\nu})\Pi_V(q^2) = i\int d^4x e^{iqx}\langle \mathrm{vac}|J_V^\mu(x)J_V^\nu(0)|\mathrm{vac}\rangle$$

$J_V^\mu = \sum_f Q_f\bar{\psi}_f\gamma^\mu\psi_f$, and its Adler function ($Q^2 = -q^2$)

$$\mathcal{A}(Q^2) \equiv -Q^2\frac{d}{dQ^2}\Pi_V(Q^2) = Q^2\int_0^\infty dt\frac{1}{(t+Q^2)^2}\frac{1}{\pi}\mathrm{Im}\Pi_V(t)\,.$$

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1) Quark-hadron duality.

Can we describe $\mathrm{Im}\Pi_V(s)$ with perturbation theory? In principle perturbation theory only applies to Euclidean quantities.

Minkowski (physical) cut (large N_c)

$$Im\Pi_V(s) \sim R(s) \sim \sum_n F_n^2 \delta(s - M_n^2)$$

This does not look like perturbation theory ...

$$Im\Pi_V^{pert.}(s) \sim constant$$

Large $N_c \rightarrow$ example of maximal duality violation (Shifman et al.)

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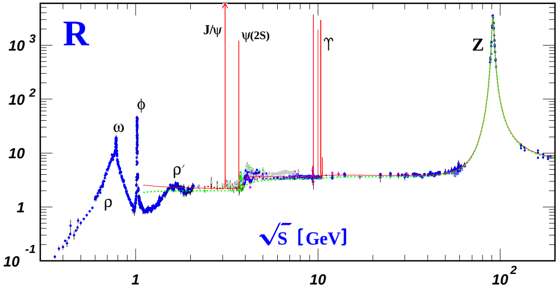
$N_c \neq \infty$ ($N_c = 3$). We expect that "in some way" we will have smoothing to the perturbative curve.
Actually this is what we see from experiment (though actually not that easy to quantify the "in some way" agreement) but this is not a "proof" that one can do perturbation theory in the Minkowski cut.



Therefore, from the mathematical point of view, one should
1) Proof that one can do perturbation theory in the Minkowski cut at finite N_c up to terms that vanishes when $q^2 \to \infty$
2) Quantify the error associated to doing the OPE in the Minkowski cut. What is left? What is the difference between OPE and the full theoretical result?

Try to be more rigorous. Only consider Euclidean quantities, where the OPE should apply. Therefore, we have to use dispersion relations.

This brings out the question: Can we trust the operator product expansion?

2) Validity of the OPE itself (Wilson):

$$\hat{A}(x)\hat{B}(0) = \sum_n C_n(x)\hat{O}_n(0)\,.$$

$\hat{O}_n$ are local operators with increasing dimensionality in n and the coefficients C_n can be computed at weak coupling.

We formulate the question in the following way: Do ALL asymptotically free gauge theories admit an OPE? The answer is NO.

We find a counterexample: QCD in two dimensions and in the large N_c limit. The 't Hooft model.

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QCD_{1+1} in the light front

$$\mathcal{L}_{1+1} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \sum_i \bar{\psi}_i \left(i\gamma^\mu D_\mu - m_i + i\epsilon \right) \psi_i \,,$$

where $D_\mu = \partial_\mu + igA_\mu$ and i labels the flavor.
x^+ =constant $\rightarrow$ quantization frame ("time")
P^- $\rightarrow$ Hamiltonian
P^+ kinetic variable.
$A^+ = 0$ gauge fixing, A^- integrated out

$$\mathcal{L} = \sum_i \psi^\dagger_{i+} i\partial^- \psi_{i+} + i \sum_i \frac{m_i^2 - i\epsilon}{4} \int dy^- \psi^\dagger_{i+}(x^-, x^+) \epsilon(x^- - y^-) \psi_{i+}(y^-, x^+)$$
$$+ \sum_{ij} \frac{g^2}{4} \int dy^- \psi^\dagger_{i+} t^a \psi_{i+}(x^-, x^+) |x^- - y^-| \psi^\dagger_{j+} t^a \psi_{j+}(y^-, x^+) \,,$$

$$P^- = -i \sum_i \frac{m_i^2 - i\epsilon}{4} \int dx^- dy^- \psi^\dagger_{i+}(x^-, x^+) \epsilon(x^- - y^-) \psi_{i+}(y^-, x^+)$$
$$- \sum_{ij} \frac{g^2}{4} \int dx^- dy^- \psi^\dagger_{i+} t^a \psi_{i+}(x^-, x^+) |x^- - y^-| \psi^\dagger_{j+} t^a \psi_{j+}(y^-, x^+) \,.$$

Large N_c

$$|ij;n\rangle = |ij;n\rangle^{(0)}$$
$$+\sum_{m,n'}\sum_k |ik;n'\rangle^{(0)}|kj;m\rangle^{(0)(0)}\langle ik;n'|^{(0)}\langle kj;m|P^-|ij;n\rangle^{(0)}\frac{1}{P_n^{(0)-}-P_m^{(0)-}-P_{n'}^{(0)-}}$$
$$+O\left(\frac{1}{N_c}\right),$$

Leading order in $1/N_c$

$$|ij;n\rangle^{(0)} = \frac{1}{\sqrt{N_c}}\int_0^{P_n^+}\frac{dp^+}{\sqrt{2(2\pi)}}\phi_n^{ij}\left(\frac{p^+}{P_n^+}\right)a^\dagger_{i,\alpha}(p)b^\dagger_{j,\alpha}(P_n-p)|0\rangle\,,$$

By applying the operator P^- to its eigenstate $|n\rangle$ at leading order in $1/N_c$ one obtains the 't Hooft equation ($x = P^+/P_n^+$)

$$M_n^2\phi_n^{ij}(x) = \hat{P}^2\phi_n^{ij}(x) \equiv \left(\frac{m_{i,R}^2}{x}+\frac{m_{j,R}^2}{1-x}\right)\phi_n^{ij}(x) - \beta^2\int_0^1 dy\,\phi_n^{ij}(y)\mathrm{P}\frac{1}{(y-x)^2}\,,$$

$$M_n^2 \simeq n\pi^2\beta^2 \qquad n\to\infty\,, \qquad m_{i,R} = m_i - \beta^2 \qquad \beta^2 = \frac{g^2N_c}{2\pi}$$

't Hooft model. The vacuum polarization

$$\mathcal{A}_X^{hadr.} = Q^2 \sum_{n_X \ldots}^{\infty} \frac{F_X^2(n)}{(M_n^2 + Q^2)^2}$$

Input ($m_{i,R}^2 = m_i^2 - \beta^2$ and $\beta^2 = g^2 N_c/(2\pi)$):

$$M^2(n) = \pi^2 \beta^2 n + (m_{i,R}^2 + m_{j,R}^2) \ln n + \cdots$$

$$\mathcal{A}_X^{OPE} = \frac{N_c}{2\pi} \left(1 + \frac{\beta^2}{Q^2} + D_X \frac{m_i m_j}{Q^2} \left(\ln\left(\frac{m_i^2}{Q^2}\right) + \ln\left(\frac{m_j^2}{Q^2}\right)\right) \right.$$
$$\left. + \frac{2\pi}{N_c} D_X \frac{m_j \langle \bar{\psi}_i \psi_i \rangle + m_i \langle \bar{\psi}_j \psi_j \rangle}{Q^2} \right) + \cdots ,$$

where $D_S = 1$ and $D_P = -1$, and we have neglected terms of $\mathcal{O}(m^2/Q^2)$ (except for the logarithm term) and terms of $\mathcal{O}(1/Q^3)$.
Output ($F^2(n) = F_{S/P}^2(n)$ for n odd/even)

$$F^2(n) = N_c \pi \beta^2 \left[1 + \frac{m_{i,R}^2 + m_{j,R}^2}{n\pi^2\beta^2} + \frac{2 m_i m_j}{n\pi^2\beta^2}(-1)^n \right] .$$

Using the Euler-MacLaurin asymptotic expansion ($\Lambda_{\rm QCD} \ll n^* \Lambda_{\rm QCD} \ll Q$)

$$\mathcal{A}(Q^2) = Q^2 \sum_{n=0}^{n^*} \frac{F_X^2(n)}{(Q^2 + M_X^2(n))^2} + Q^2 \sum_{n=n^*}^{\infty} \frac{F_X^2(n)}{(Q^2 + M_X^2(n))^2} .$$

Using the Euler-MacLaurin asymptotic expansion ($\Lambda_{\rm QCD} \ll n^* \Lambda_{\rm QCD} \ll Q$)

$$\mathcal{A}(Q^2) = Q^2 \int_0^\infty dn \frac{F_X^2(n)}{(Q^2 + M_X^2(n))^2} + \left[\sum_{n=0}^{n^*-1} \frac{Q^2 F_X^2(n)}{(Q^2 + M_X^2(n))^2} - \int_0^{n^*} \frac{Q^2 F_X^2(n)}{(Q^2 + M_X^2(n))^2} \right]$$

$$+ \frac{Q^2}{2} \frac{F_X^2(n^*)}{(Q^2 + M_X^2(n^*))^2} + Q^2 \sum_{k=1}^{\infty} (-1)^k \frac{|B_{2k}|}{(2k)!} \frac{d^{(2k-1)}}{dn^{(2k-1)}} \frac{F_X^2(n)}{(Q^2 + M_X^2(n))^2} \bigg|_{n=n^*},$$

Using the Euler-MacLaurin asymptotic expansion ($\Lambda_{\rm QCD} \ll n^* \Lambda_{\rm QCD} \ll Q$)

$$\mathcal{A}(Q^2) \simeq Q^2 \int_0^\infty dn \frac{F_X^2(n)}{(Q^2 + M_X^2(n))^2} + \cdots$$

This is the single term that produces $\ln Q^2$

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This is the single term that produces $\ln Q^2$

$$\int_{n^*}^\infty dn \frac{1}{n^s} \frac{1}{(Q^2 + nB)^{2+r}} \ln^t n$$

$$r \to \frac{1}{Q^{2r}} \qquad s \to \frac{1}{Q^{2s}} \left(\ln^s Q^2 + \cdots \right) \qquad t \to \ln^s Q^2 + \cdots$$

We get a systematic method to get corrections to the decay constant for a given spectrum.

$$F_S(n) = \frac{m}{2\sqrt{\pi}} \int_0^1 dx \phi_n(x) \left(\frac{1}{x} - \frac{1}{1-x} \right) = \frac{m}{\sqrt{\pi}} \int_0^1 dx \frac{\phi_n(x)}{x} \text{ for } n \text{ odd}$$

and zero otherwise. For the pseudoscalar we have

$$F_P(n) = \frac{m}{2\sqrt{\pi}} \int_0^1 dx \phi_n(x) \left(\frac{1}{x} + \frac{1}{1-x} \right) = \frac{m}{\sqrt{\pi}} \int_0^1 dx \frac{\phi_n(x)}{x} \text{ for } n \text{ even},$$

$$\int_0^1 dx \frac{\phi_n^{ij}(x)}{x} = \pi \frac{\beta}{m_i} \left[1 + \frac{m_{i,R}^2 + m_{j,R}^2}{2n\pi^2\beta^2} + \frac{m_i m_j}{n\pi^2\beta^2}(-1)^n + \mathcal{O}\left(\frac{1}{n^2}\right) \right].$$

By using symmetries and the 't Hooft equation, we can also obtain

$$M^2(n) \int_0^1 dx \phi_n^{ij}(x) = \pi\beta m_i \left[1 + \frac{m_{i,R}^2 + m_{j,R}^2}{2n\pi^2\beta^2} + \frac{m_i m_j}{n\pi^2\beta^2}(-1)^n \right]$$
$$+(-1)^n \pi\beta m_j \left[1 + \frac{m_{i,R}^2 + m_{j,R}^2}{2n\pi^2\beta^2} + \frac{m_i m_j}{n\pi^2\beta^2}(-1)^n \right] + \mathcal{O}\left(\frac{1}{n^2}\right),$$

Numerical check

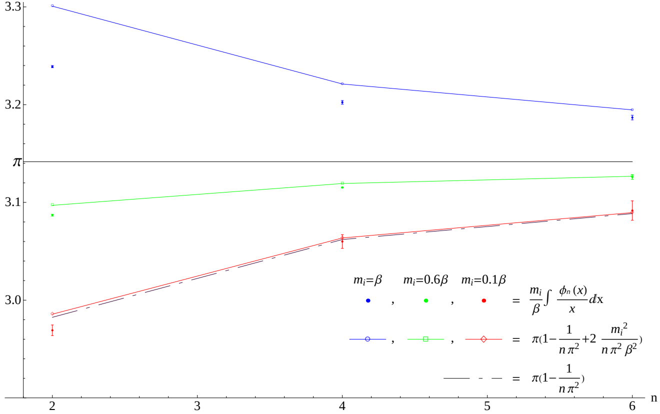

Figure: Equal mass case.

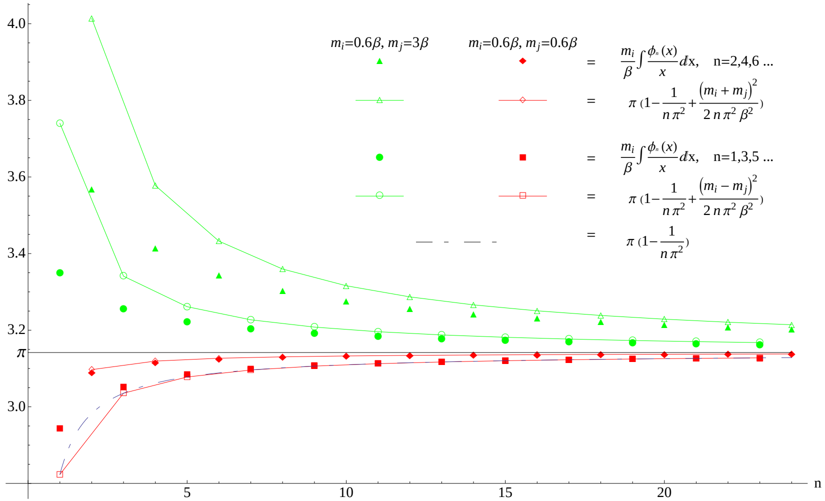

Figure: Non-equal mass case.

CONCLUSION: OPE works fine for the vacuum polarization.

Deep Inelastic Scattering

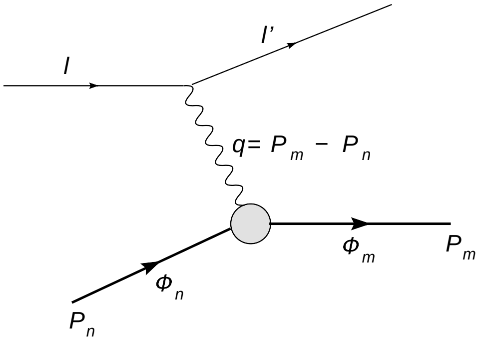

Figure: Deep-inelastic scattering off a light meson.

$$T^{\mu\nu}(q) = i\int d^2x e^{iq\cdot x}\langle ij;n|Tj^{\mu}(x)j^{\nu}(0)|ij;n\rangle$$
$$\equiv \left(P_n^{\mu} - \frac{q^{\mu}q\cdot P_n}{q^2}\right)\left(P_n^{\mu} - \frac{q^{\mu}q\cdot P_n}{q^2}\right)T(Q^2,x_B)\,,$$

where $x_B = Q^2/(2P_n\cdot q)$, $Q^2 = -q^2$, P_n is the moment of the meson and i, j stand for the flavor of the quark and antiquark, respectively, which form the bound state. $j^{\mu}(x) = \sum_h j_h^{\mu}(x)$, where $j_h^{\mu}(x) = e_h\bar{\psi}_h\gamma^{\mu}\psi_h(x)$.
The imaginary part of $T^{\mu\nu}$ is proportional to the differential cross section when $x_B \geq 0$, and in the light-cone frame reads

$$\mathrm{Im}\,T^{++} = \frac{1}{2}\sum_m\int\frac{dP_m^+}{2(2\pi)P_m^+}\left|\langle ij;m|j^+(0)|ij;n\rangle\right|^2(2\pi)^2\delta^2(q+P_n-P_m)$$
$$= \quad \pi\sum_m\left|\langle ij;m|\sum_{h=i,j}e_h j_h^+(0)|ij;n\rangle\right|^2\delta\left(M_m^2 - M_n^2 - Q^2\frac{(1-x_B)}{x_B}\right)\,.$$

$$\langle ij;m|\bar{\psi}_i\gamma^-\psi_i|ij;n\rangle = -2\pi\beta\frac{m_i}{q^+}\left(\left[1-m_i\frac{m_i+m_j(-1)^m}{Q^2}+\frac{m_{i,R}^2+m_{j,R}^2}{2Q^2}\frac{x}{1-x}\right.\right.$$
$$\left.+\frac{m_im_j}{Q^2}\frac{x}{1-x}(-1)^m\right]\phi_n^{ij}(x)$$
$$\left.+\frac{x}{Q^2}(m_{i,R}^2+m_im_j(-1)^m)\phi_n^{ij\prime}(x)\right)+o\left(\frac{1}{Q^2}\right)$$

$$\langle ij;m|\bar{\psi}_i\gamma^\mu\psi_i|ij;n\rangle = \left(P_n^\mu+P_m^\mu+\frac{(\mu_n^2-\mu_m^2)}{q^2}q^\mu\right)P_{mn}^{ij,i}(q^2)\,,$$
$$\langle ij;m|\bar{\psi}_j\gamma^\mu\psi_j|ij;n\rangle = \left(P_n^\mu+P_m^\mu+\frac{(\mu_n^2-\mu_m^2)}{q^2}q^\mu\right)A_{mn}^{ij,j}(q^2)\,.$$

$P_{nm}^{ij,i}(q^2)$ and $A_{nm}^{ij,j}(q^2)$ are related by charge symmetry:

$$A_{nm}^{ij,j}(q^2)=-(-1)^{n+m}P_{nm}^{ji,j}(q^2)\,.$$

When $1-x \gg \beta^2/Q^2$

$$
\mathrm{Im}\,T \simeq 4\pi\left(\frac{2\pi\beta x}{Q^2}\right)^2 \frac{1}{\left(1+x^2\frac{M_n^2}{Q^2}\right)^2}\sum_{m=0}^{\infty}\delta\left(M_m^2-M_n^2(1-x)-Q^2\frac{(1-x)}{x}\right)
$$

$$
\times\left[e_i m_i\left\{\phi_n^{ij}(x)\left(1-\frac{m_i^2+m_i m_j(-1)^m}{Q^2}\right.\right.\right.
$$

$$
+\left(\frac{m_{i,R}^2+m_{j,R}^2}{2Q^2}+\frac{m_i m_j}{Q^2}(-1)^m\right)\frac{x}{1-x}\Bigg)
$$

$$
\left.\left.+\frac{x}{Q^2}(m_{i,R}^2+m_i m_j(-1)^m)\frac{d\phi_n^{ij}(x)}{dx}\right)\right\}
$$

$$
-(-1)^m e_j m_j\left\{\phi_n^{ij}(1-x)\left(1-\frac{m_j^2+m_j m_i(-1)^m}{Q^2}\right.\right.
$$

$$
+\left(\frac{m_{j,R}^2+m_{i,R}^2}{2Q^2}+\frac{m_j m_i}{Q^2}(-1)^m\right)\frac{x}{1-x}\Bigg)
$$

$$
\left.\left.\left.+\frac{x}{Q^2}(m_{j,R}^2+m_j m_i(-1)^m)\frac{d\phi_n^{ij}(1-x)}{dx}\right)\right\}+o\left(\frac{1}{Q^2}\right)\right]^2 .
$$

Using dispersions relations, one should have

$$T(Q^2, x_B) = \frac{2}{\pi}\int_0^{x_B^{max}} dy_B \frac{1}{y_B}\frac{\mathrm{Im}\, T(Q^2, y_B)}{1-\left(\frac{y_B}{x_B}\right)^2 - i\epsilon},$$

where $x_B^{max} = 1/(1+(P_0^2 - P_n^2)/Q^2)$. Therefore, $T(Q^2, x_B)$ admits an analytic expansion in $1/x_B$ for $x_B > x_B^{max}$,

$$T(Q^2, x_B) = 4\sum_{N=0,2,4,\dots} M_N(Q^2)\frac{1}{x_B^N},$$

where

$$M_N(Q^2) \equiv \frac{1}{2\pi}\int_0^{x_B^{max}} dy_B y_B^{N-1}\mathrm{Im}\, T(Q^2, y_B).$$

$$x_B = \frac{1}{1+\frac{P_m^2 - P_n^2}{Q^2}} = \frac{x}{1 - x^2\frac{P_n^2}{Q^2}}$$

$x = -q^+/P_n^+$, $x_{max} = 1 - M_0^2/Q^2 + \mathcal{O}(1/Q^4)$

$$M_N = \sum_{m=0}^{\infty} [\cdots]$$

Euler-MacLaurin expansion

$$\sum_m f(m) = \int dm f(m) + \cdots$$

$$(-1)^m \to \frac{1}{Q^{2+\beta_i}} \leftarrow (-1)^{\frac{Q^2}{\pi^2\beta^2}\frac{1-x}{x}}$$

$$(-1)^{2m} = 1!! \to \frac{1}{Q^2} \leftarrow (-1)^{2\frac{Q^2}{\pi^2\beta^2}\frac{1-x}{x}}$$

$$M_m^2 \simeq \pi^2\beta^2 m \simeq Q^2\frac{(1-x)}{x}$$

$$M_N(Q^2) \simeq M_N^{Eul.}(Q^2) \equiv \frac{8}{Q^4}\int_0^{x_{max}} dx \left(\frac{x}{1-\frac{M_n^2}{Q^2}x^2}\right)^N \frac{x}{1-\frac{M_n^4}{Q^4}x^4}$$
$$\times \left[e_i^2 m_i^2 \left(\phi_n^{ij}(x)\right)^2 + e_j^2 m_j^2 \left(\phi_n^{ij}(1-x)\right)^2\right.$$
$$+2e_i^2 m_i^2 \phi_n^{ij}(x)\left[-\frac{m_i^2}{Q^2}\phi_n^{ij}(x) + x\frac{m_{i,R}^2}{Q^2}\frac{d\phi_n^{ij}(x)}{dx}\right]$$
$$+2e_j^2 m_j^2 \phi_n^{ij}(1-x)\left[-\frac{m_j^2}{Q^2}\phi_n^{ij}(1-x) + x\frac{m_{j,R}^2}{Q^2}\frac{d\phi_n^{ij}(1-x)}{dx}\right]$$
$$\left.-4e_ie_jm_i^2m_j^2\frac{2x-1}{Q^2(1-x)}\phi_n^{ij}(x)\phi_n^{ij}(1-x) - 2e_ie_jm_i^2m_j^2\frac{x}{Q^2}\frac{d}{dx}\left(\phi_n^{ij}(x)\phi_n^{ij}(1-x)\right)\right]$$

$$T^{Eul.}(Q^2,x_B) = 4\sum_{N=0,2,4,\dots} M_N^{Eul.}(Q^2)\frac{1}{x_B^N} = \frac{2}{\pi}\int_0^1 \frac{dy_B}{y_B}\frac{\mathrm{Im}\,T^{Eul.}(Q^2,y_B)}{1-\left(\frac{y_B}{x_B}\right)^2 - i\epsilon},$$

Note that $\mathrm{Im}\, T^{Eul.} \neq \mathrm{Im}\, T$.

$$
\begin{aligned}
(\mathrm{Im})T^{Eul.}(Q^2, x_B) \;&=\; -2\left(\frac{4}{Q^2}\right)^2 \int_{-\infty}^{\infty} dy \left\{ e_i^2 (\mathrm{Im}) J_i(x,y) f_i(y) \right. \\
&\left. + e_j^2 (\mathrm{Im}) J_j(x,y) f_j(y) + e_i e_j (\mathrm{Im}) J_{int.}(x,y) f_{int.}(y) \right\} ,
\end{aligned}
$$

where

$$
f_i(y) \equiv \left[\phi_n^{ij}(y)\right]^2 = \int_{-\infty}^{\infty} \frac{dx^-}{2(2\pi)} e^{-iyP_n^+ \frac{x^-}{2}} \langle ij; n|\psi^\dagger_{i,+}(x^-)\psi_{i,+}(0)|ij; n\rangle ,
$$

$$
f_j(y) \equiv \left[\phi_n^{ij}(1-y)\right]^2 = -\int_{-\infty}^{\infty} \frac{dx^-}{2(2\pi)} e^{-iyP_n^+ \frac{x^-}{2}} \langle ij; n|\psi^\dagger_{j,+}(0)\psi_{j,+}(x^-)|ij; n\rangle ,
$$

$$
f_{int.}(y) \equiv \frac{m_i m_j}{y(1-y)} \phi_n^{ij}(y)\phi_n^{ij}(1-y) = \frac{(P_n^+)^2}{N_c} \int_{-\infty}^{\infty} \frac{dx^-}{2(2\pi)} e^{-iyP_n^+ \frac{x^-}{2}} \int_{-\infty}^{\infty} dz^-
$$

$$
\times \langle ij; n|\psi^\dagger_{i,-}(x^-)\psi_{j,-}(z^-)\psi^\dagger_{j,+}(0)\psi_{i,+}(z^-)|ij; n\rangle .
$$

The functions J are defined as

$$J_i(x,y) \equiv \frac{m_i^2 x^2 y}{y^2 - x^2 + i\epsilon}\left[1 - 2\frac{m_i^2}{Q^2} - 2\frac{M_n^2}{Q^2}y^2 + \frac{m_{i,R}^2}{Q^2}y\frac{d}{dy}\right],$$

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$$J_{int.}(x,y) \equiv 2\frac{m_i m_j}{Q^2}\left[x^2\frac{2y^2(1-2y)}{y^2 - x^2 + i\epsilon} - x^3\frac{d}{dx}\frac{y^2(1-y)}{y^2 - x^2 + i\epsilon}\right].$$

Perturbative factorization

We can now perform the same computation using the OPE.

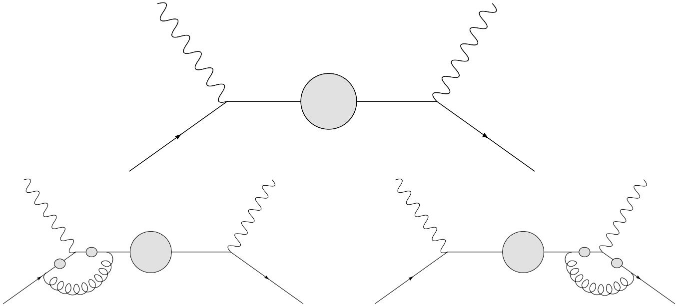

Figure: *Diagrams contributing to the perturbative computation at $O(\beta^2/Q^2)$.*

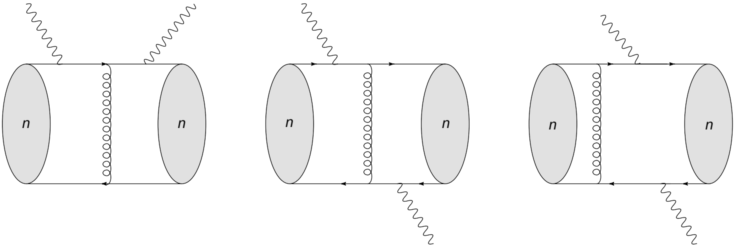

Figure: *Additional β^2 corrections.*

We obtain

$$T^{OPE}(Q^2, x_B) = -2\left(\frac{4}{Q^2}\right)^2 \int_{-\infty}^{\infty} dy \left\{ e_i^2 J_i(x,y) f_i(y) + e_j^2 J_j(x,y) f_j(y) \right\} .$$

We can easily see that $T^{OPE} \neq T^{Eul}$. If we consider the moments generated by T^{OPE}, M_N^{OPE}, they can be expressed in terms of non-perturbative local operators as expected. Nevertheless, M_N has extra terms. If we consider the difference, we obtain

$$M_N(Q^2) - M_N^{OPE}(Q^2) = \frac{16 e_i e_j}{Q^4} \frac{1}{N_c} \frac{m_i m_j}{Q^2} \int dz^- \langle ij; n| \psi_{i,-}^\dagger(0) \frac{(-i\overleftarrow{D}^+)^{N+2}}{(P_n^+)^{N+1}}$$
$$\times \left[(N+4)\left(1 - \frac{-i\overleftarrow{D}^+}{P_n^+}\right) - 2\frac{-i\overleftarrow{D}^+}{P_n^+} \right] \psi_{j,-}(z^-) \psi_{j,+}^\dagger(0) \psi_{i,+}(z^-) |ij; n\rangle ,$$

which is expressed in terms of non-local operators. We take this result as evidence of the existence of OPE-violating terms.

Conclusions

Usually stated: "QCD is an asymptotically free gauge theory. Therefore, we can use perturbation theory to compute $\sigma(e^+e^- \to hadrons)$".
This statement is false. Large N_c analysis give a general counterexample and the 't Hooft model an specific one.
Usually stated: "QCD is an asymptotically free gauge theory. Therefore, we can use perturbation theory to compute the OPE".
This statement is false. The 't Hooft model gives an specific counterexample:

$$i \int d^2x e^{iq\cdot x} T j^\mu(x) j^\nu(0) \stackrel{?}{=} \sum_n C_n(Q^2) \hat{O}_n(0) \,.$$

This equality should hold independently of the asymptotic states.

$$\langle vac| i \int d^2x e^{iq\cdot x} T j^\mu(x) j^\nu(0) |vac\rangle \stackrel{?}{=} \sum_n C_n(Q^2) \langle vac|\hat{O}_n(0)|vac\rangle \,.$$

The answer is yes for the vacuum polarization with the present precision.

$$\langle ij;n| i \int d^2x e^{iq\cdot x} T j^\mu(x) j^\nu(0) |ij;n\rangle \stackrel{?}{=} \sum_n C_n(Q^2) \langle ij;n|\hat{O}_n(0)|ij;n\rangle \,.$$

The answer is NO for DIS.

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We have computed the moments of DIS both from the OPE and by direct hadronic computation and we have found disagreement. This signals a violation of the OPE. The first time ever.
We expect a similar phenomena to happen in four dimensions in the large N_c limit.

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