NUCLEAR EFFECTS IN ATOMIC PHYSICS FROM EFFECTIVE THEORIES

ANTONIO PINEDA
(IFAE, UAB)

hep-ph/0210210 (PRC67 (2003) 025201)
hep-ph/0310135 (NPA737 (2004) S298)
hep-ph/0412142 (PRC71 (2005) 065205)
Nevado & Pineda (Preliminary)

•) HADRONIC/PARTICLE PHYSICS

ATOMIC PHYSICS ⟷ NUCLEAR PHYSICS

•) Precise measurements in atomic physics ⟶ Learning about nuclear structure

•) Hyperfine splitting ((muonic) hydrogen) → Nature (1972)

$E_{HF}^{exp} = E(n=1, s=1) - E(n=1, s=0)$

Total spin

$\nu_{HF} = \frac{E_{HF}}{h} = 14\underset{O(\alpha)}{\underline{2}}0.\underset{O(\alpha^2)}{\underline{4}}057517667(9)$ MHz (12 digits)

$\nu_{HF}(\text{QED}) = 1420.4\underline{5}195(14)$

proton static source

anomalous dimension

$\delta V = \frac{4\pi\alpha(1+\mu_p)}{3 m_p m_e} \zeta^2 \delta(\vec{r})$

$O(\alpha^2)$

vs

$O\left(\alpha \frac{m_e}{m_N}\right)$

Large numerical factor

$\langle E(n=2, l=1) - E(n=2, l=0) \rangle_s$

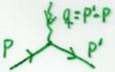
•) $\boxed{\text{Lamb shift}}$ ((muonic) hydrogen) → PSI

Neutrino factory

$\boxed{\text{Proton radius}}$

·) Definition of the proton (neutron) radius

$$\langle p',s|J^\mu|p,s\rangle = \bar{u}(p')\left[F_1(q^2)\gamma^\mu + i F_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$



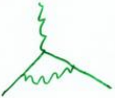
$$J^\mu = \sum_i Q_i \bar{q}_i \gamma^\mu q_i$$

$$F_i(q^2) = F_i + \frac{q^2}{m^2}F_i' + \dots$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$\boxed{r_p^2 = 6\left.\frac{dG_{p,E}(q^2)}{dq^2}\right|_{q^2=0}}$$

Infrared divergent !!

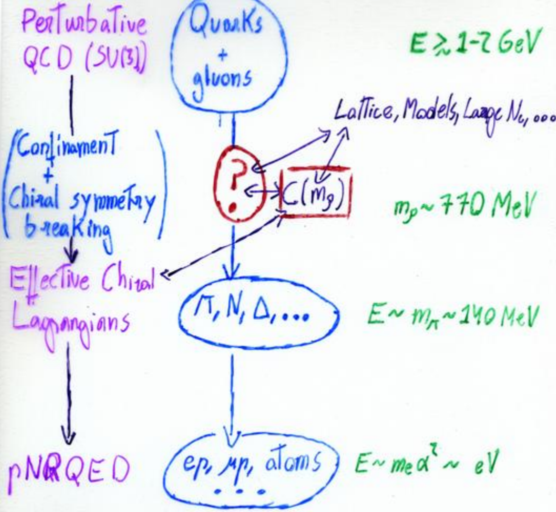


"proton radius" $\longrightarrow$ "matching coefficient"

$$r_p \longrightarrow r_p(\nu)$$

r_p is a function of the matching coefficients of the effective theory

Particle Physics versus Nuclear Physics versus Atomic Physics



$$\mathrm{HBET} \xrightarrow[m_\pi]{} \mathrm{QED} \xrightarrow[m_\ell]{} \mathrm{NRQED} \xrightarrow[m_\ell \alpha]{} \mathrm{pNRQED} \quad (E \sim m_\ell \alpha^2)$$

Integrating out $\rightarrow m_\pi$

HBET $(E \sim m_\pi)$

$\frac{m_\pi}{M_N}, \frac{m_\pi}{4\pi F_\pi}, \frac{m_\pi}{m_\tau} \ll 1$

$$\mathcal{L}_{HBET} = \mathcal{L}_\gamma + \mathcal{L}_\pi + \mathcal{L}_N + \mathcal{L}_{N\ell} + \dots$$

$$\mathcal{L}_\gamma = -\frac{1}{4} F^2 + \boxed{\left(\frac{d_{2,R}}{m_p^2} + \frac{d_2^{(\tau)}}{m_\tau^2}\right) F_{\mu\nu} D^2 F^{\mu\nu}} + \dots$$

$$\mathcal{L}_\pi = \frac{F_\pi^2}{4} \mathrm{Tr}[(D_\mu U) D^\mu U^\dagger] + \dots \qquad U = u^2 = e^{i\frac{\Pi}{F_\pi}}$$

$$\mathcal{L}_N = N^\dagger (i v^\mu \nabla_\mu + g_A u_\mu S^\mu) N + \dots + (\mathrm{DELTA}) +$$

$$\nabla_\mu = \partial_\mu + \Gamma_\mu \qquad u_\mu = i u^\dagger (\nabla_\mu U) u$$

$$\Gamma_\mu = \frac{1}{2}\{u^\dagger(\partial_\mu + ieQA_\mu)u + u(\partial_\mu + ieQA_\mu)u^\dagger\}$$

$$+ \boxed{N_p^\dagger \left\{-e \frac{c_D^{(p)}}{m_p^2}[\vec{\nabla}\cdot\vec{E}]\right\} N_p}$$

$$\mathcal{L}_{N\ell} = \boxed{\frac{1}{m_p^2} \sum_i c_{3,R}^{\ell_i} \bar{N}_p \gamma^0 N_p \bar{\ell}_i \gamma^0 \ell_i} +$$

$$+ \frac{1}{m_p^2} \sum_i c_{4,R}^{\ell_i} \bar{N}_p \gamma^j \gamma_5 N_p \bar{\ell}_i \gamma_j \gamma_5 \ell_i$$

(NRQED) $E \sim (\alpha) m_\ell$

$$\mathcal{L}_{NRQED} = -\frac{1}{4} F^2 + \boxed{\frac{d_{2,NR}}{m_p^2} F_{\mu\nu} D^2 F^{\mu\nu}} + \dots$$

$$+ N_p^\dagger \left(i D_0^{(p)} + \frac{\vec{D}_{(p)}^2}{2m} + \dots \right) N_p$$

$$+ \boxed{N_p^\dagger \left\{ -e \frac{c_D^{(p)}}{m_p^2} [\vec{\nabla} \cdot \vec{E}] \right\} N_p}$$

(QED) $\rightarrow$ $$+ \boxed{N_p^\dagger \left\{ C_{A_1}^{(p)} e^2 \frac{\vec{B}^2 - \vec{E}^2}{8 m_p^3} - C_{A_2}^{(p)} e^2 \frac{\vec{E}^2}{16 m_p^3} \right\} N_p}$$

$$+ \boxed{\frac{c_{3,NR}^{p\ell}}{m_p^2} N_p^\dagger N_p \ell^\dagger \ell} - \frac{c_{4,NR}^{p\ell}}{m_p^2} N_p^\dagger \vec{\sigma} N_p \ell^\dagger \vec{\sigma} \ell$$

$$C_{A_1}^{(p)} = 4 m_p^3 \frac{\beta_M^{(p)}}{\alpha}$$

$$C_{A_2}^{(p)} = -\frac{8 m_p^3}{\alpha} \left(\underline{\underline{\alpha_E^{(p)}}} + \underline{\underline{\beta_M^{(p)}}} \right)$$

Proton polarizabilities

Matching (getting the radius from the lamb shift)

$$d_{2,NR} = d_{2,R} + \frac{m_p^2}{4}\Pi'_{h,n}(0) = \frac{m_p^2}{4}\Pi'_h(0)$$

$$C_{3,NR} = i g^4 m_p m_\ell \int \frac{d^4K}{(2\pi)^4}\frac{1}{K^4}\frac{1}{K^4 - 4m_\ell^2 K_0^2} \times$$

$$\times\left\{S_1(K_0,K^2)(-3K_0^2+\vec{K}^2) - \vec{K}^2 S_2(K_0,K^2)\right\}$$

$$T^{\mu\nu}(q) = i\int d^4x\, e^{iqx}\langle p,s|T\{J^\mu(x)J^\nu(0)\}|p,s\rangle =$$

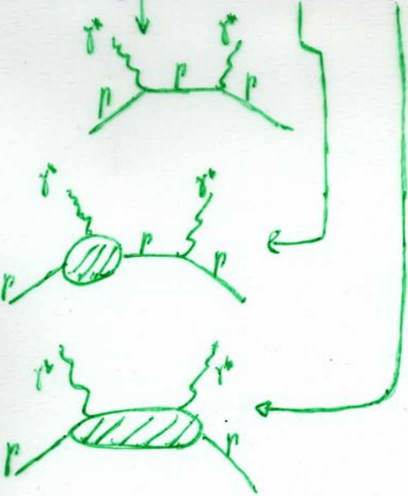
$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)S_1(\rho,q^2) + \frac{1}{m_p^2}\left(p^\mu - \frac{m_p\rho}{q^2}q^\mu\right)\left(p^\nu - \frac{m_p\rho}{q^2}q^\nu\right)S_2(\rho,q^2) + \ldots$$

$$C_{3,NR} = C_{3,R} + \delta C_{3,\text{point-like}} + \delta C_{3,\text{Zemach}} + \delta C_{3,\text{pol}}$$

$= C \times$

HBET NRQED

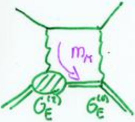
$$T^{\mu\nu}(q) = T^{\mu\nu}_{\text{point-like}} + T^{\mu\nu}_{\text{Zemach}} + T^{\mu\nu}_{\text{pol}}$$



$\cdot)\ \delta C_{3,\text{point-like}} = \frac{m_p}{m_\ell}\left(\ln\frac{m_\ell^2}{\nu^2}+\frac{1}{3}\right)\alpha^2$

$\cdot)\ \delta C_{3,\text{Zemach}} = 4(4\pi\alpha)^2 m_p^2 m_\ell \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \frac{1}{k^6} \overset{1}{\overset{\parallel}{G_E^{(0)}}} G_E^{(2)} =$

$$= 2(\pi\alpha)^2 \left(\frac{m_p}{4\pi F_0}\right)^2 \frac{m_\ell}{m_\pi}\left\{\frac{3}{4}g_A^2+\frac{1}{8}+\frac{2}{\pi}g_{\pi N\Delta}^2\frac{m_\pi}{\Delta}\sum_{n=0}^{\infty} C_n\left(\frac{m_\pi}{\Delta}\right)^{2n}+ g_{\pi N\Delta}^2\sum_{n=1}^{\infty} H_n\left(\frac{m_\pi}{\Delta}\right)^{2n}\right\}$$



Dimensional regularization!!

$\cdot)\ \delta C_{3,\text{pol}}(\text{logs}) = -\alpha m_p^2 m_\ell \left[5\alpha_E^{(p)} - \beta_M^{(p)}\right] \ln m_\ell =$

$$= -\frac{2}{9}\alpha^2\frac{m_\ell}{\Delta}b_{1,F}^2\ln\frac{\Delta}{m_\ell}+\frac{49}{12}\pi\alpha^2 g_A^2\frac{m_\ell}{m_\pi}\frac{m_p^2}{(4\pi F_0)^2}\ln\left(\frac{m_\pi}{m_\ell}\right)+$$

$$+\frac{8}{27}\alpha^2 g_{\pi N\Delta}^2\frac{m_\ell}{\sqrt{\Delta^2-m_\pi^2}}\frac{m_p^2}{(4\pi F_0)^2}\left(\frac{45\Delta}{\sqrt{\Delta^2-m_\pi^2}}+\frac{4\Delta^2-49m_\pi^2}{\Delta^2-m_\pi^2}\ln R\right)\ln\left(\frac{m_\pi}{m_\ell}\right)$$

$(m_\ell \ll m_\pi)$

$$C_n = \frac{(-1)^n\,\Gamma(-3/2)}{\Gamma(n+1)\,\Gamma(-\frac{3}{2}-n)}\left\{B_{6+2n} - \frac{2(n+2)}{3+2n}B_{4+2n}\right\}$$

$$B_n = \int_0^\infty dt\,\frac{t^{2-n}}{\sqrt{1-t^2}}\ln\left[\frac{1}{t}+\sqrt{\frac{1}{t^2}-1}\right]$$

$$H_n = \frac{n!\,(2n-1)!!\,\Gamma(-\frac{3}{2})}{2(2n)!!\,\Gamma(\frac{1}{2}+n)}$$

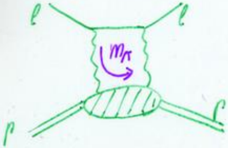
(PRELIMINARY)

$$\delta C_{3,pol} = -(4\pi\alpha)^2 m_p \frac{m_\ell}{m_\pi}\left[4 m_p \left(\frac{g_A}{2F_\pi}\right)^2\right] \int \frac{d^{D-1}K}{(2\pi)^{D-1}} \frac{1}{(1+\vec{P}_1^2)^4} \cdot$$

$$\cdot \int_0^\infty \frac{dw}{\pi} w^{D-5} \frac{1}{w^2 + \frac{m_\ell^2}{m_\pi^2}\frac{2}{(1+\vec{P}_1^2)^2}} \left\{ \left(2+(1+\vec{P}_1^2)^2\right) A_E + (1+\vec{P}_1^2)^2 \vec{P}_1^2 w^2 B_E \right\}$$

$$A_E = -\frac{1}{4\pi}\left\{ \sqrt{1+w^2} + \int_0^1 dx \frac{1-x}{\sqrt{1+x(1-x)w^2(1+\vec{P}_1^2)+x^2w^2}} - \frac{3}{2} \right\}$$

$$B_E = \frac{1}{8\pi}\left\{ \int_0^1 dx \frac{1-2x}{\sqrt{1+x(1-x)w^2(1+\vec{P}_1^2)+x^2w^2}} - \frac{1}{2}\int_0^1 dx \frac{(1-x)(1-2x)^2}{\left(\sqrt{1+x(1-x)w^2(1+\vec{P}_1^2)+x^2w^2}\right)^3} \right\}$$



·) Phenomenological parameterization of the form factors

$$G_E^{ph}(\vec{q}^{\,2}) = \frac{1}{\left(1+\frac{\vec{q}^{\,2}}{\Lambda^2}\right)^2} \qquad G_M^{ph}(\vec{q}^{\,2}) = \mu\, G_E^{ph}(\vec{q}^{\,2})$$

Incorrect chiral structure

pNRQED ($E \sim m_\ell \alpha^2$)

$$\mathcal{L}_{pNRQED} = S^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_\ell} + V^{(0)} + \dots \boxed{\delta V} \right) S + \dots$$

$S(\vec{X}, \vec{x})$ $\qquad V^{(0)} = -Z_p Z_\ell \frac{\alpha}{r}$

Wave function (field) representing the atom

$$\delta V = \frac{D_d^{had}}{m_p^2} \delta^{(3)}(\vec{r})$$

$$\boxed{D_d^{had} = -c_{3,NR}^{p\ell_i} - 16\pi\alpha Z_{\ell_i} Z_p d_{2,NR} + \frac{\pi\alpha}{2} Z_{\ell_i} c_D^{(p)}}$$

$$\delta E = \langle E(s) - E(p) \rangle \doteq \frac{D_d^{had}}{m_p^2} \delta_{\ell 0} \frac{1}{\pi} \left(\frac{\mu_{\ell p}\alpha}{n} \right)^3$$

Hyperfine

$$\delta V = 2 \frac{c_{4,NR}}{m_p^2} \vec{S}^2 \delta^{(3)}(\vec{r})$$

$$\delta E = 4 \frac{c_{4,NR}}{m_p^2} \frac{1}{\pi} (\mu_{\ell p}\alpha)^3$$

NUMBERS

$$\delta E^{\text{zemach}}_{ep,\Delta\to\infty} \simeq -\frac{14.5773}{n^3}\ \text{Hz}$$

$$\delta E^{\text{zemach}}_{ep} \simeq -\frac{27.9772}{n^3}\ \text{Hz}$$

$$\delta E^{\text{pol.}}_{ep,\Delta\to\infty}(\log) \simeq -\frac{64.4841}{n^3}\ \text{Hz} \to -\frac{91.0303}{n^3}\ \text{Hz (COMPLETE)}$$

$$\delta E^{\text{pol}}_{ep}\ (\log) \simeq -\frac{77.6037}{n^3}\ \text{Hz}$$

$$\delta E^{\text{zemach}}_{\mu p,\Delta\to\infty} \simeq -\frac{0.08064}{n^3}\ \text{meV}$$

$$\delta E^{\text{zemach}}_{\mu p} \simeq -\frac{0.15376}{n^3}\ \text{meV} = -\frac{1}{n^3}(0.08064 + 0.04488 + \dots)\ \text{meV}$$

$$\delta E^{\text{pol}}_{\mu p,\Delta\to\infty} \simeq -\frac{0.167477}{n^3}\ \text{meV (COMPLETE)}$$

$$\delta E^{\text{vac. pol.}}_{ep} \simeq -\frac{3.39990}{n^3}\ \text{kHz}$$

$$\delta E^{\text{vac. pol.}}_{\mu p} \simeq -\frac{0.09039}{n^3}\ \text{meV}$$

Definition of the proton (neutron) radius

- $$\boxed{r_p^2(\nu) = 6 \frac{d}{dq^2} G_{E,p}(q^2)\Big|_{q^2=0} = \frac{3}{4}\frac{1}{m_p^2}\left(C_D^{(p)}(\nu) - 1\right)}$$

$C_D^{(p)} = 1 + 2F_2 + 8F_1'$ $\quad r_p(\nu) = r_p^{(0)} + \alpha\, r_p^{(1)}(\nu) + \dots$

$\nu \frac{d}{d\nu} r_p^{(0)} = 0 \quad r_p^{(0)} \nu \frac{d}{d\nu} r_p^{(1)} = -\frac{1}{\pi}\frac{1}{m_p^2}, \dots$

- Neutron

$$r_n^2 = 6 \frac{d}{dq^2} G_{E,n}(q^2)\Big|_{q^2=0} = \frac{3}{4}\frac{1}{m_n^2} C_D^{(n)}$$

$C_D^{(n)} = 2F_2^{(n)} + 8F_1^{(n)\prime}$

$$b_{n\ell} = \frac{1}{4m_n}\left(\alpha Z_\ell C_D^{(n)} - \frac{3}{\pi} C_{3,NR}^{n\ell}\right) \sim D_d^{\text{had}\,(n)}$$

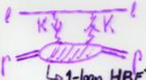
neutron-lepton scattering length

It is not proportional to the radius!

(real) low energy constant

Hyperfine Splitting

$$C_{4,NR} = -i\frac{g^4}{3}\int\frac{d^D K}{(2\pi)^D}\,\frac{1}{K^2}\,\frac{1}{K^4-4m_p^2K_0^2}\Big\{A_1(K_0,K^2)(K_0^2+2K^2)+ 3K^2\frac{K_0}{m_p}A_2(K_0,K^2)\Big\}$$



1-loop HBET

off-shell photons

$$T^{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\langle p,s|T\{J^\mu(x)J^\nu(0)\}|p,s\rangle$$

$$= -\frac{i}{m_p}\epsilon^{\mu\nu\rho\sigma}\Big(q_\rho S_\sigma A_1(\nu,q^2) + \frac{q_\rho}{m_p^2}\big((m_p\nu)S_\sigma-(q\cdot S)p_\sigma\big)A_2(\nu,q^2)\Big)$$

$$\boxed{C_{4,NR} = C_{4,R} + \delta C_{4,\text{point-like}} + \delta C_{4,\text{Zemach}} + \delta C_{4,\text{pol}}}$$

$$\delta C^{u,d}_{4,\text{Zemach}} = (4\pi\alpha)^2 m_p\frac{2}{3}\int\frac{d^{D-1}K}{(2\pi)^{D-1}}\frac{1}{K^4}G_E^{(0)}G_M^{(2)}$$

$$= \frac{m_p^2}{(4\pi F_\pi)^2}\alpha^2\frac{2}{3}\pi^2 g_A^2\ln\frac{m_\pi^2}{\nu^2}$$

$m_p\frac{\#}{(4\pi F_\pi)^2}|\vec{K}|$

e

$m_p >> |\vec{K}| >> m_\pi$

non-analytical behavior in $|\vec{K}|^2$

$\sqrt{\vec{K}^2}$

HBET

$A_i \rightarrow A_i^{pole} + \bar{A}_i$ (Ji-Osborne)

Zemach

polarizability

$$\delta C_{4,NR}(\nu) = \left(1 - \frac{m_\ell^2}{4}\right)\alpha^2 \ln \frac{m_\ell^2}{\nu^2}$$

point-like

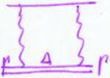
Zemach

$$+ \frac{m_p^2}{(4\pi F_0)^2}\alpha^2 \frac{2}{3}\pi^2 g_A^2 \ln \frac{m_\pi^2}{\nu^2}$$

$$+ \frac{m_p^2}{(4\pi F_0)^2}\alpha^2 \frac{8}{27}\pi^2 g_{N\pi\Delta}^2 \ln \frac{\Delta^2}{\nu^2}$$

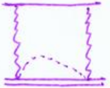
polarization

$$+ \frac{b_{1,F}^2}{18}\alpha^2 \ln \frac{\Delta^2}{\nu^2}$$



$$- \frac{m_p^2}{(4\pi F_0)^2} g_A^2 \frac{\alpha^2}{\pi}\frac{8}{3} C \ln \frac{m_\pi^2}{\nu^2}$$

$$+ \frac{m_p^2}{(4\pi F_0)^2} g_{\pi N\Delta}^2 \frac{\alpha^2}{\pi}\frac{64}{27} C \ln \frac{\Delta^2}{\nu^2}$$



SU(2)

Zemach

$$+ \frac{m_p^2}{(4\pi F_0)^2}\alpha^2 \frac{2}{9}\pi^2 (-5D^2 + 6DF - 9F^2) \ln \frac{m_K^2}{\nu^2}$$

$+ (\Delta\text{'s})$ (?)

+ (pol. SU(3)) (?)

SU(3)

$$C = 2\int_0^1\int_0^1 dy\, dx\, \sqrt{1-y^2}\Big(-2x(2+y^2) +$$

$$+ \frac{1}{y}\Big(2(1-x)x(2+y^2)\sqrt{\frac{1}{x-x^2+x^2y^2}}$$

$$-3(1-2x)y^2\sqrt{\frac{x}{1-x(1-y^2)}}\Big)\operatorname{Sinh}^{-1}\Big[\sqrt{\frac{x}{1-x}}\,y\Big]\Big)$$

$$= -0.165037 = \frac{\pi^3}{12} - \frac{7}{8}\pi$$

$$\boxed{\begin{aligned}\delta C_{4,NR}(\nu) = {} & \left(1-\frac{\mu_p^2}{4}\right)\alpha^2 \ln\frac{m_\ell^2}{\nu^2} + \frac{b_{1,F}^2}{18}\alpha^2 \ln\frac{\Delta^2}{\nu^2} + \\ & + \frac{m_\pi^2}{(4\pi F_0)^2}\alpha^2\frac{2}{3}\left(\frac{3}{3}+\frac{7}{2\pi^2}\right)\pi^2 g_A^2 \ln\frac{m_\pi^2}{\nu^2} \\ & + \frac{m_\pi^2}{(4\pi F_0)^2}\alpha^2\frac{8}{27}\left(\frac{5}{3}-\frac{7}{\pi^2}\right)\pi^2 g_{\pi N\Delta}^2 \ln\frac{\Delta^2}{\nu^2}\end{aligned}}$$

$$\boxed{\delta C_{4,NR}(\nu)\Big|_{N_c\to\infty} = \alpha^2 \ln\frac{m_\ell^2}{\nu^2} + \frac{m_\pi^2}{(4\pi F_0)^2}\alpha^2\pi^2 g_A^2 \ln\frac{m_\pi^2}{\nu^2}}$$

NUMBERS (Hyperfine splitting)

$$\delta V \sim C_{NR,4} \, \vec{v}_1 \cdot \vec{v}_2 \, \frac{\delta(\vec{r})}{M_N^2}$$

$$\Delta E_{HF}(QED) - \Delta E_{HF}(exp) \sim -0.046 \text{ MHz}$$

$$\Delta E_{HF}^{(had)} \sim \left[m_e \alpha^4 \frac{m_e}{M_N} \right] \underbrace{\left[\frac{m_e}{M_N} \frac{C_{NR}^{(had)}}{\alpha} \right]}_{O\left(\alpha \frac{m_e}{M_N}\right)} \sim \boxed{-0.031 \text{ MHz}}$$

$$\Rightarrow \boxed{C_{R,4}(m_\rho) \sim -16\alpha^2}$$

$$\Delta E_{HF,\pi}^{\text{Zemach}}(m_\rho) \simeq -0.022 \text{ MHz}$$

$$\Delta E_{HF,\Delta}^{\text{Zemach}}(m_\rho) \simeq -0.004 \text{ MHz}$$

$$\Delta E_{HF,\text{point-like}} \simeq -0.003 \text{ MHz}$$

$$\Delta E_{HF,\text{pol}} \simeq -0.002 \text{ MHz}$$

CONCLUSIONS (HF)

- $\boxed{\delta E_{HF} \simeq \frac{m_l^3 \alpha^5}{m_p^2} \times (\ln m_q, \ln \Delta, \ln m_{l_i})}$

- $C_R \sim \alpha^2 \times (\ln m_q, \ln \Delta, \ln m_{l_i})$

- $C_R(m_\rho) \sim -16\alpha^2$

- $\delta E_{HF}(m_\rho) \simeq -0.031$ MHz
  $\left(\sim \frac{2}{3} \times [E_{HF}(QED) - E_{HF}(exp)]\right)$

- Methodology to connect Chiral Lagrangians with atomic physics: potential NRQED

HBET $\to$ QED $\to$ NRQED $\to$ pNRQED

$m_\mu \longrightarrow m_e \longrightarrow m_e\alpha \longrightarrow m_e\alpha^2$

- Next Problem:

Lamb Shift $\Rightarrow$ proton radius

CONCLUSIONS

- Definition of the proton (neutron) radius as a matching coefficient of the effective theory

$$\boxed{r_p^2(\nu) = \frac{3}{4}\frac{1}{m_p^2}\left(c_D^{(p)}(\nu) - 1\right)}$$

- $\boxed{\delta E \sim m_\ell \alpha^5 \left(\frac{m_\ell}{m_p}\right)^2 F\left(\frac{m_\ell}{m_\pi}\right)}$ (Lamb shift)

We have computed the leading contribution in HBET (no new counterterms)

- Hydrogen (beyond present experimental accuracy)
- Muonic Hydrogen. Relevant for ongoing & future experiments